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# Investigation of the inherent trade-off between bias model complexity and state estimation accuracy in INS/GNSS-Integration

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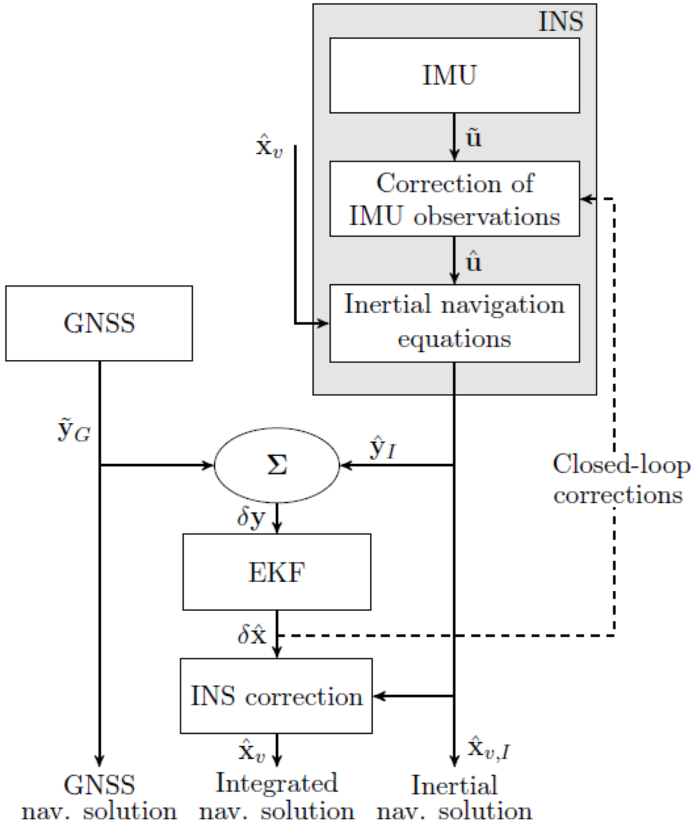
# Objectives of the investigation

- INS/GNSS integration forms the central navigation unit for many outdoor applications
- Knowledge of IMU sensor errors
- Impact of accelerometer bias modelling on the navigation solution accuracy



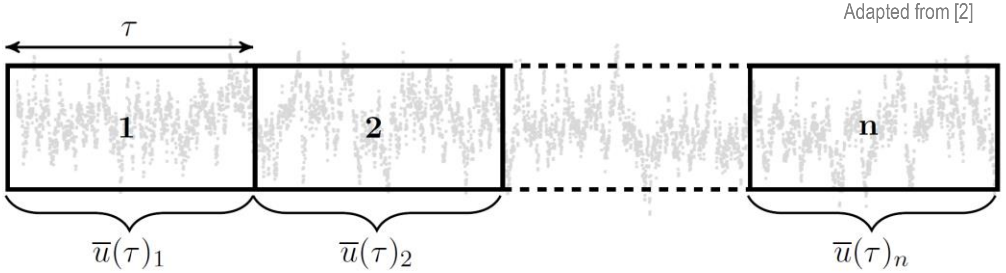
# Outline

1. Analysis of long-term IMU recordings
2. Modelling of IMU errors in a loose INS/GNSS integration architecture
3. Simulation study and results
4. Conclusions and Outlook

# Analysis of long-term IMU recordings

- Analysis based on the Allan Variance Method
- Goal: Identification and quantification of the underlying noise processes
- Analysis of a tactical grade IMU (3-axis servo-accelerometer & 3-axis fiber optic gyro)

# Allan Variance – Basic concept



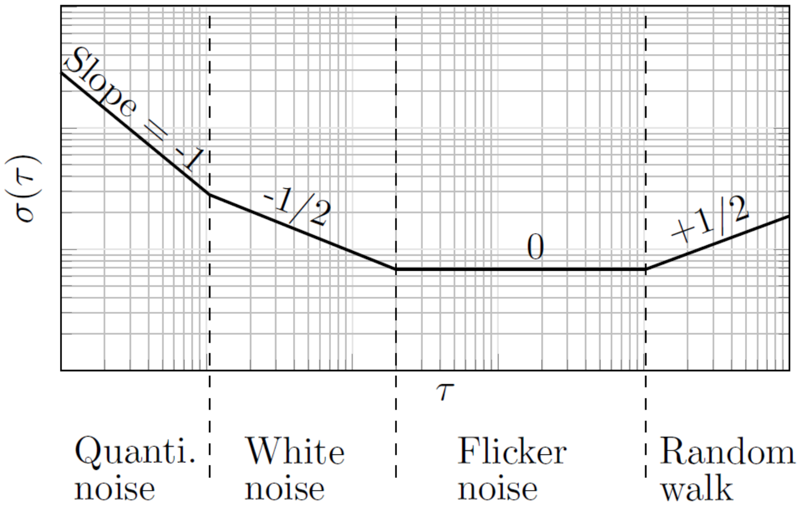
Adapted from [2]

1. Data record length: 6 hours (non-moving IMU)
2. Compute the average of each block (n … number of blocks)

   List of averages $[\bar{u}(\tau)_1 \quad \bar{u}(\tau)_2 \quad \cdots \quad \bar{u}(\tau)_n]$
3. Allan variance [1]

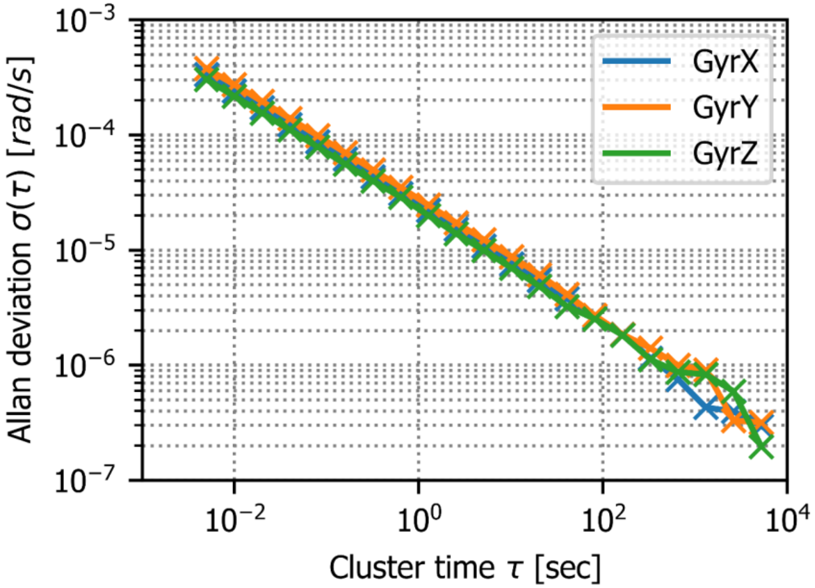
$$\sigma^2(\tau) = \frac{1}{2(n-1)} \sum_{i}^{n-1} (\bar{u}(\tau)_{i+1} - \bar{u}(\tau)_i)^2$$

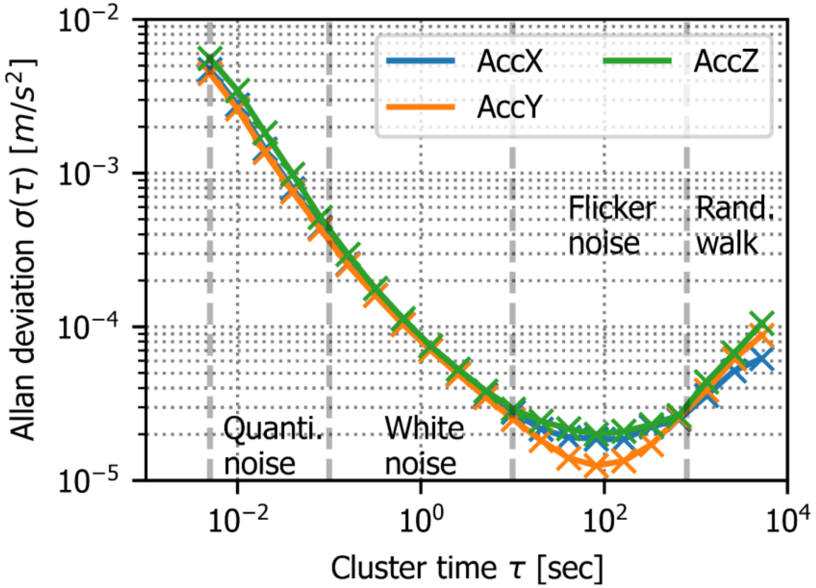
Allan deviation: $\sigma(\tau) = \sqrt{\sigma^2(\tau)}$



Adapted from [3] IEEE Standard 952

# Allan Variance – Results I




$$z_g(t) = z_{g,N}(t)$$

White noise

$$z_a(t) = z_{a,A}(t) + z_{a,N}(t) + z_{a,B}(t) + z_{a,K}(t)$$

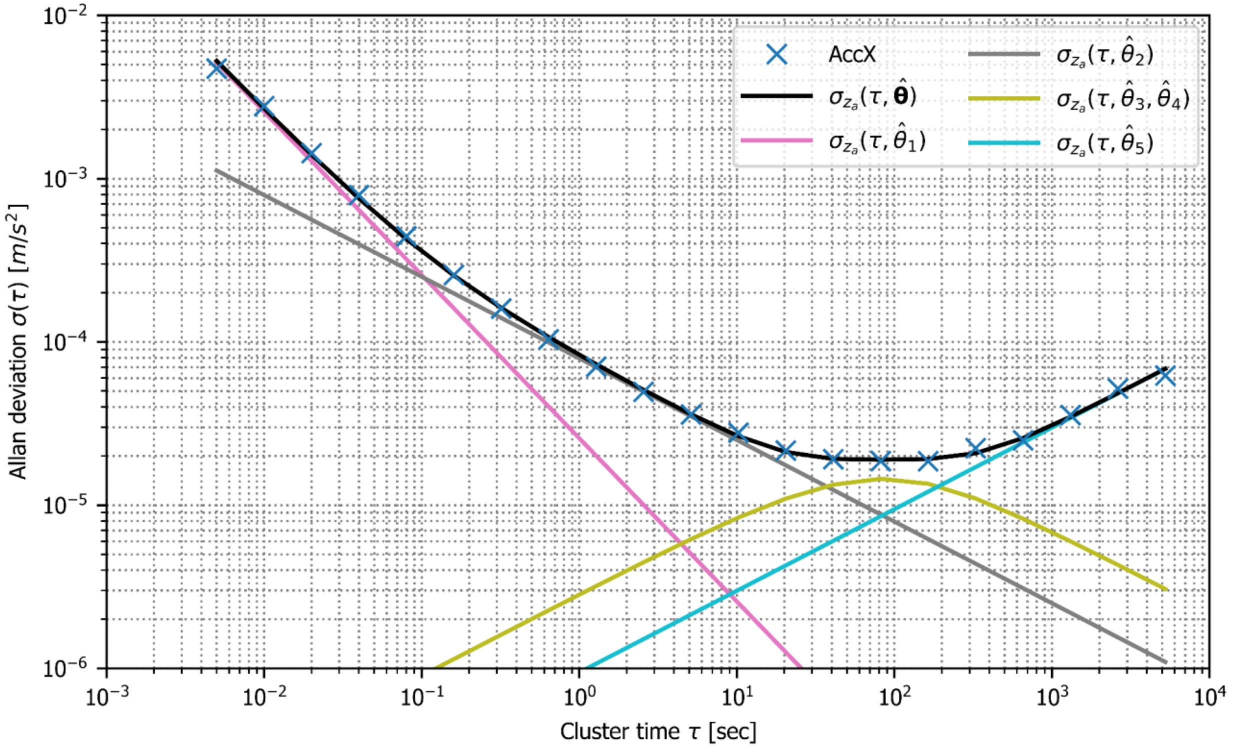
Quantization noise, White noise, Flicker noise, Random walk

# Allan Variance – Results II

- Estimating the parameters of the noise processes via a LSQ fit
- Defining the parameter vector $\boldsymbol{\vartheta}$

  $$\boldsymbol{\vartheta} = [S_{a,A} \quad S_{a,N} \quad S_{a,B} \quad T_B \quad S_{a,K}]$$

- First order Gauß Markov (FOGM) process is used to approximate the flicker noise process



# Modelling of IMU errors in a loose INS/GNSS integration architecture

- IMU observation model

$$\tilde{\boldsymbol{f}} = \boldsymbol{f} + \boldsymbol{b}_a + \boldsymbol{z}_{a,N}$$

$$\tilde{\boldsymbol{\omega}} = \boldsymbol{\omega} + \boldsymbol{b}_g + \boldsymbol{z}_{g,N}$$

- IMU bias model

$$\boldsymbol{b}_a = \boldsymbol{b}_{a,0} + \boldsymbol{z}_{a,B} + \boldsymbol{z}_{a,K}$$

$$\boldsymbol{b}_g = \underbrace{\boldsymbol{b}_{g,0}}_{\text{static biases}}$$

# Modelling of IMU errors in a loose INS/GNSS integration architecture

- Structure of the classical system model [4]:

$$\underbrace{\begin{bmatrix}\delta\dot{\boldsymbol{\Psi}}^n\\ \delta\dot{\boldsymbol{v}}^n\\ \delta\dot{\boldsymbol{r}}^n\\ \delta\dot{\boldsymbol{b}}_a\\ \delta\dot{\boldsymbol{b}}_g\end{bmatrix}}_{\delta\dot{\boldsymbol{x}}} = \underbrace{\begin{bmatrix} \boldsymbol{F}_{\Psi\Psi} & \boldsymbol{F}_{\Psi v} & \boldsymbol{F}_{\Psi r} & \boldsymbol{0}_3 & \boldsymbol{C}_b^n \\ \boldsymbol{F}_{v\Psi} & \boldsymbol{F}_{vv} & \boldsymbol{F}_{vr} & \boldsymbol{C}_b^n & \boldsymbol{0}_3\\ \boldsymbol{F}_{r\Psi} & \boldsymbol{F}_{rv} & \boldsymbol{F}_{rr} & \boldsymbol{0}_3 & \boldsymbol{0}_3\\ \boldsymbol{0}_3 & \boldsymbol{0}_3 & \boldsymbol{0}_3 & \boldsymbol{0}_3 & \boldsymbol{0}_3\\ \boldsymbol{0}_3 & \boldsymbol{0}_3 & \boldsymbol{0}_3 & \boldsymbol{0}_3 & \boldsymbol{0}_3\end{bmatrix}}_{\boldsymbol{F}} \underbrace{\begin{bmatrix}\delta\boldsymbol{\Psi}^n\\ \delta\boldsymbol{v}^n\\ \delta\boldsymbol{r}^n\\ \delta\boldsymbol{b}_a\\ \delta\boldsymbol{b}_g\end{bmatrix}}_{\delta\boldsymbol{x}} + \underbrace{\begin{bmatrix}\boldsymbol{C}_b^n & \boldsymbol{0}_3\\ \boldsymbol{0}_3 & \boldsymbol{C}_b^n\\ \boldsymbol{0}_3 & \boldsymbol{0}_3\\ \boldsymbol{0}_3 & \boldsymbol{0}_3\\ \boldsymbol{0}_3 & \boldsymbol{0}_3\end{bmatrix}}_{\boldsymbol{G}} \underbrace{\begin{bmatrix}\boldsymbol{w}_{g,N}\\ \boldsymbol{w}_{a,N}\end{bmatrix}}_{\boldsymbol{w}}$$

- Structure of the detailed system model:

$$\begin{bmatrix}\delta\dot{\boldsymbol{\Psi}}^n\\ \delta\dot{\boldsymbol{v}}^n\\ \delta\dot{\boldsymbol{r}}^n\\ \delta\dot{\boldsymbol{b}}_a\\ \delta\dot{\boldsymbol{b}}_g\end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_{\Psi\Psi} & \boldsymbol{F}_{\Psi v} & \boldsymbol{F}_{\Psi r} & \boldsymbol{0}_3 & \boldsymbol{C}_b^n \\ \boldsymbol{F}_{v\Psi} & \boldsymbol{F}_{vv} & \boldsymbol{F}_{vr} & \boldsymbol{C}_b^n & \boldsymbol{0}_3\\ \boldsymbol{F}_{r\Psi} & \boldsymbol{F}_{rv} & \boldsymbol{F}_{rr} & \boldsymbol{0}_3 & \boldsymbol{0}_3\\ \boldsymbol{0}_3 & \boldsymbol{0}_3 & \boldsymbol{0}_3 & -\boldsymbol{I}_3 T_B^{-1} & \boldsymbol{0}_3\\ \boldsymbol{0}_3 & \boldsymbol{0}_3 & \boldsymbol{0}_3 & \boldsymbol{0}_3 & \boldsymbol{0}_3\end{bmatrix} \begin{bmatrix}\delta\boldsymbol{\Psi}^n\\ \delta\boldsymbol{v}^n\\ \delta\boldsymbol{r}^n\\ \delta\boldsymbol{b}_a\\ \delta\boldsymbol{b}_g\end{bmatrix} + \begin{bmatrix}\boldsymbol{C}_b^n & \boldsymbol{0}_3 & \boldsymbol{0}_3 & \boldsymbol{0}_3\\ \boldsymbol{0}_3 & \boldsymbol{C}_b^n & \boldsymbol{0}_3 & \boldsymbol{0}_3\\ \boldsymbol{0}_3 & \boldsymbol{0}_3 & \boldsymbol{0}_3 & \boldsymbol{0}_3\\ \boldsymbol{0}_3 & \boldsymbol{0}_3 & \boldsymbol{I}_3 & \boldsymbol{I}_3\\ \boldsymbol{0}_3 & \boldsymbol{0}_3 & \boldsymbol{0}_3 & \boldsymbol{0}_3\end{bmatrix} \begin{bmatrix}\boldsymbol{w}_{g,N}\\ \boldsymbol{w}_{a,N}\\ \boldsymbol{w}_{a,B}\\ \boldsymbol{w}_{a,K}\end{bmatrix}$$

# Modelling of IMU errors in a loose INS/GNSS integration architecture

- Classical system noise VCM: $$\boldsymbol{Q}_{k-1} = \boldsymbol{T}_{k-1}\boldsymbol{G}_{k-1}\begin{bmatrix}\boldsymbol{I}_3 S_{g,N} & \\ & \boldsymbol{I}_3 S_{a,N}\end{bmatrix}\boldsymbol{G}_{k-1}^T\boldsymbol{T}_{k-1}^T\Delta t$$

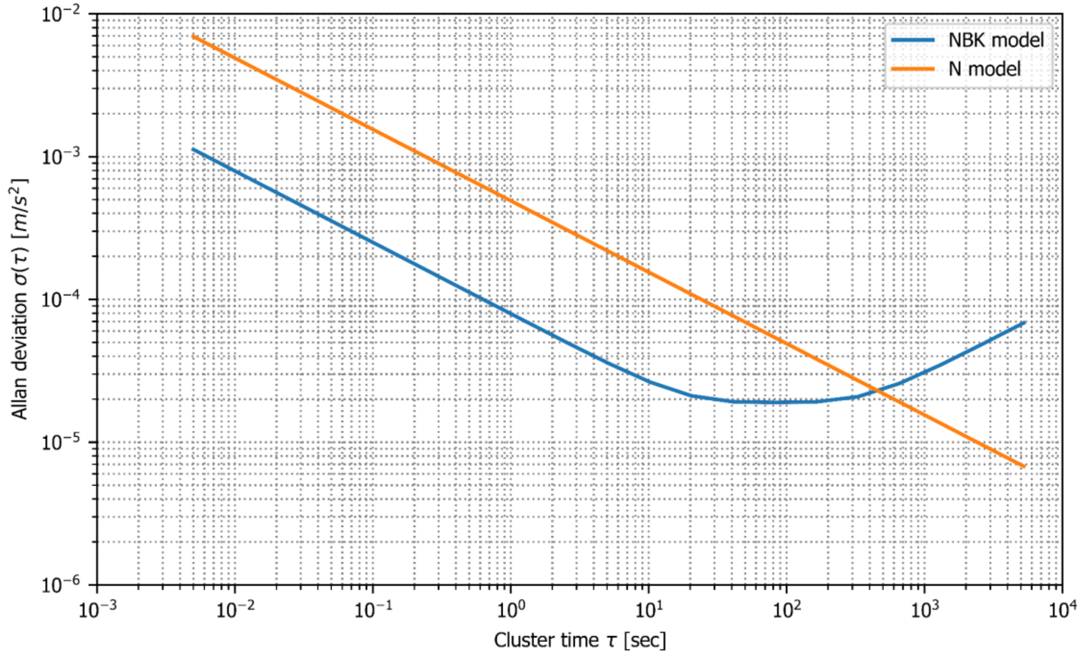
- Detailed system noise VCM: $$\boldsymbol{Q}'_{k-1} = \boldsymbol{T}'_{k-1}\boldsymbol{G}'_{k-1}\begin{bmatrix}\boldsymbol{I}_3 S_{g,N} & & & \\ & \boldsymbol{I}_3 S_{a,N} & & \\ & & \boldsymbol{I}_3 S_{a,B} & \\ & & & \boldsymbol{I}_3 S_{a,K}\end{bmatrix}\boldsymbol{G}'^T_{k-1}\boldsymbol{T}'^T_{k-1}\Delta t$$

# Simulation study

1. Classical modeling approach via WN processes (N model)
   – Applying the manufacturer WN specification
2. Detailed modeling approach via WN, FOGM and RW (NBK model)
   – Applying the estimated noise parameters

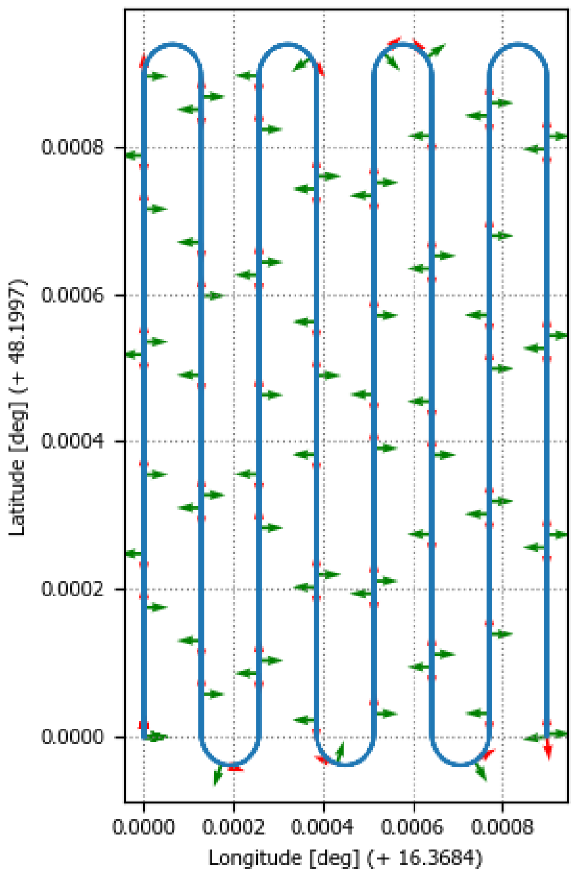
The two modeling approaches are compared for two cases:

1. Continuous GNSS coverage (1 Hz)
2. GNSS signal outage over a period of 5 minutes

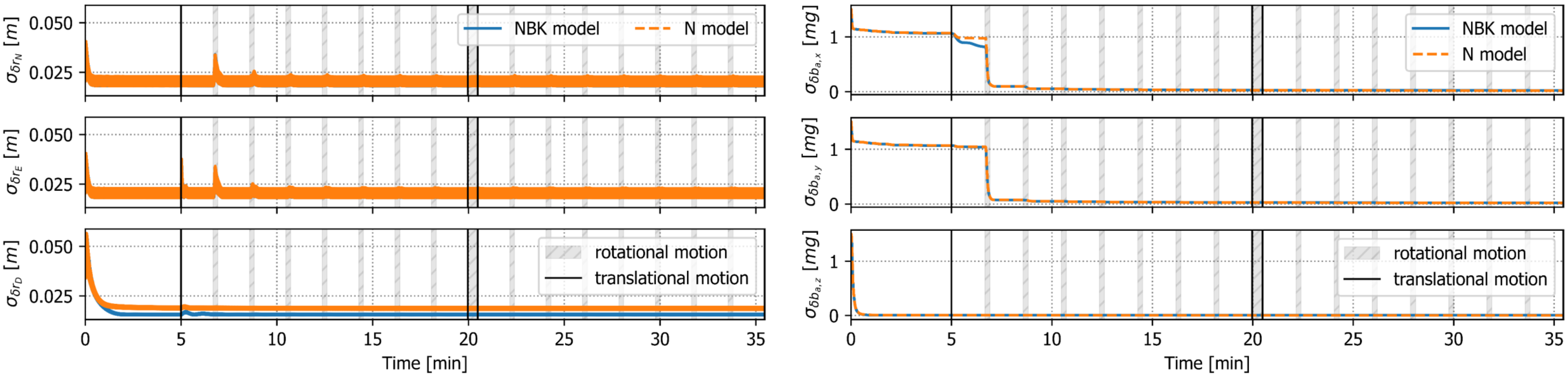


# Simulation study – Motion scenario

- True IMU and GNSS observations are determined from the simulated motion scenario
- Generate sensor errors:
  - RTK precision for the GNSS observation errors
  - Replicate an IMU with identical stochastic properties as those of the IMU investigated (except quantization noise)

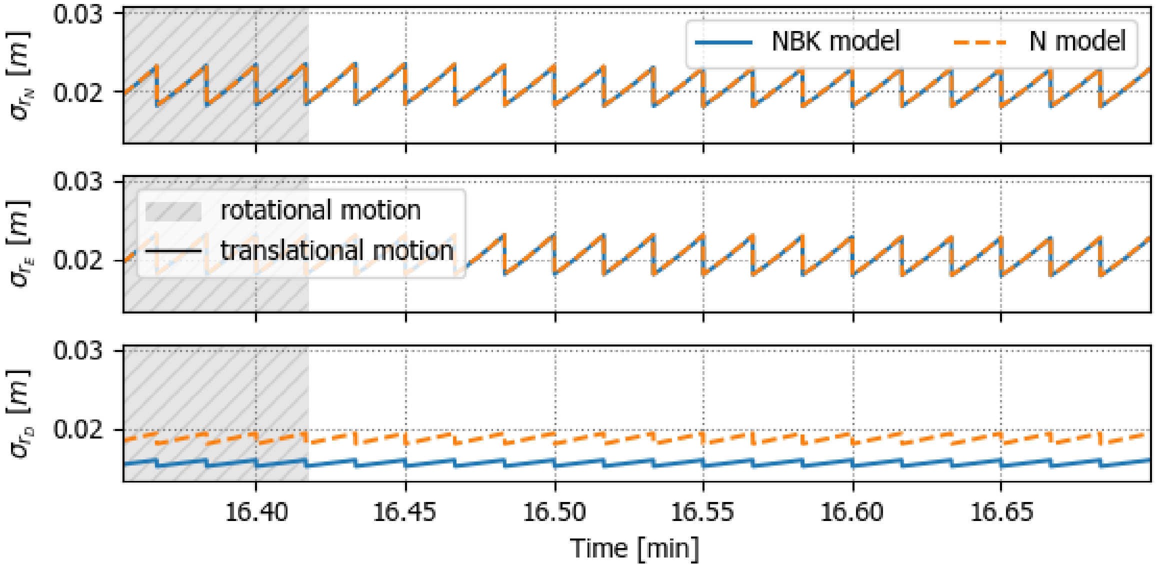


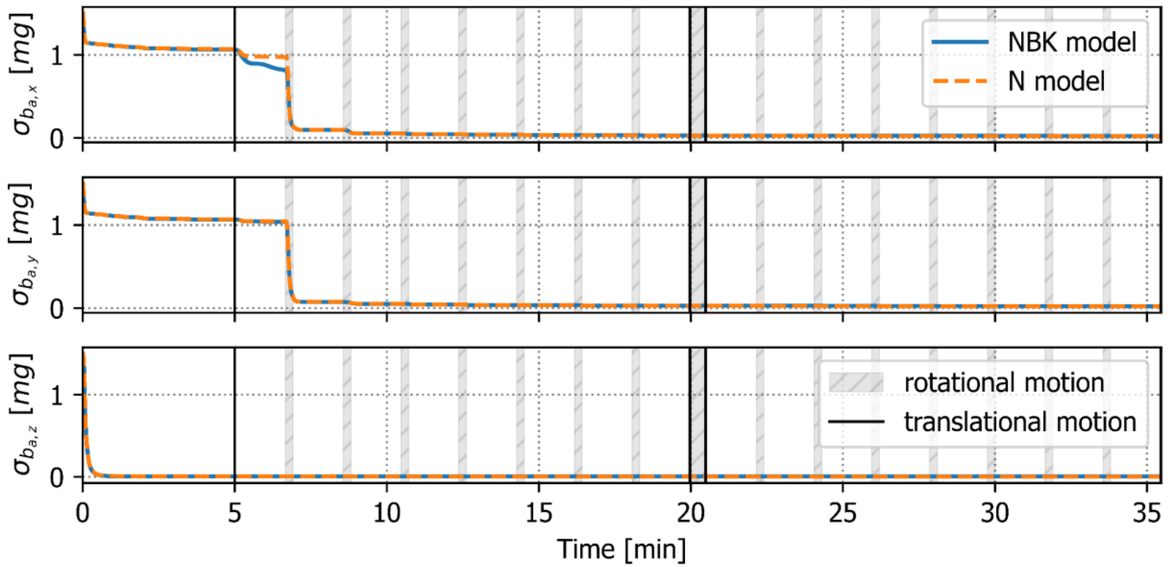
# Simulation results – Case 1 (No GNSS outages)




White noise of NBK model: $8.2\ \mu g/\sqrt{\mathrm{Hz}}$
White noise of N model: $50\ \mu g/\sqrt{\mathrm{Hz}}$

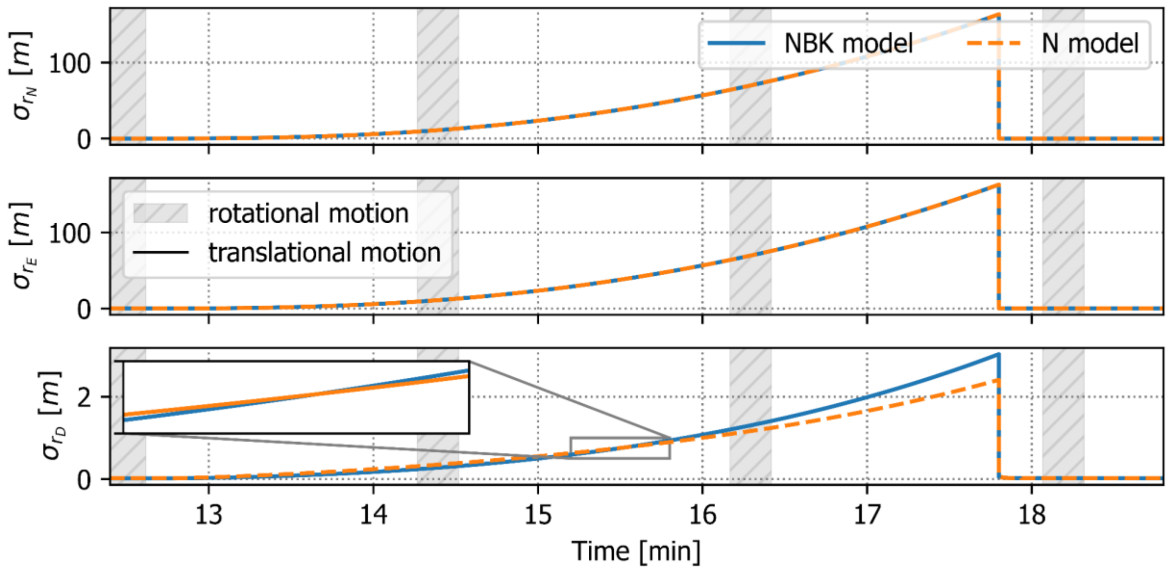
# Simulation results – Case 1 (No GNSS outages)

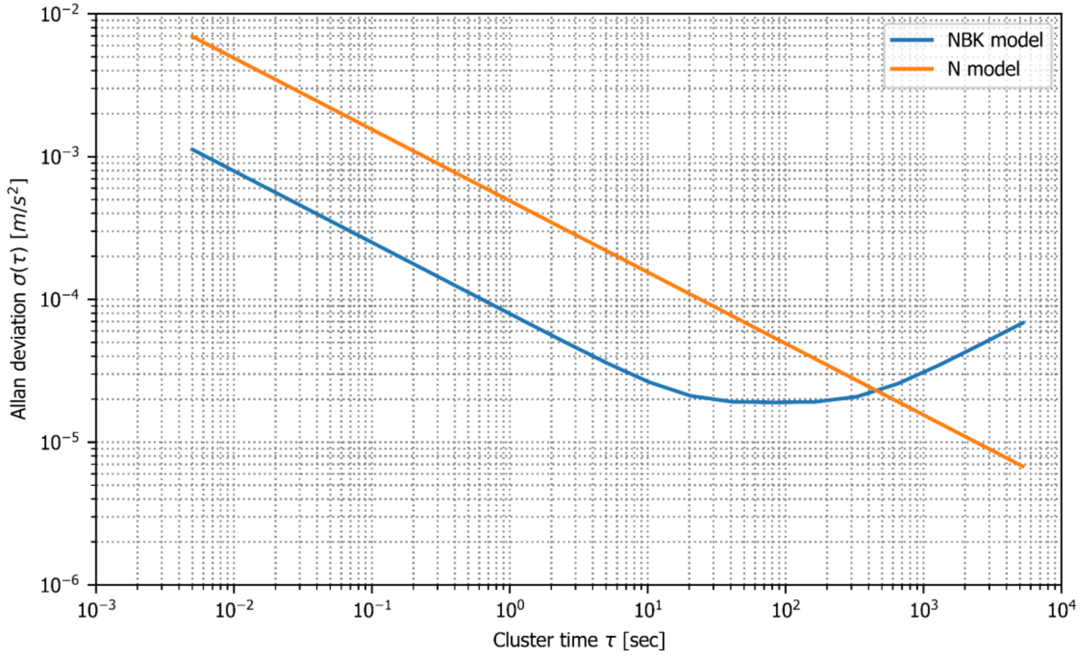


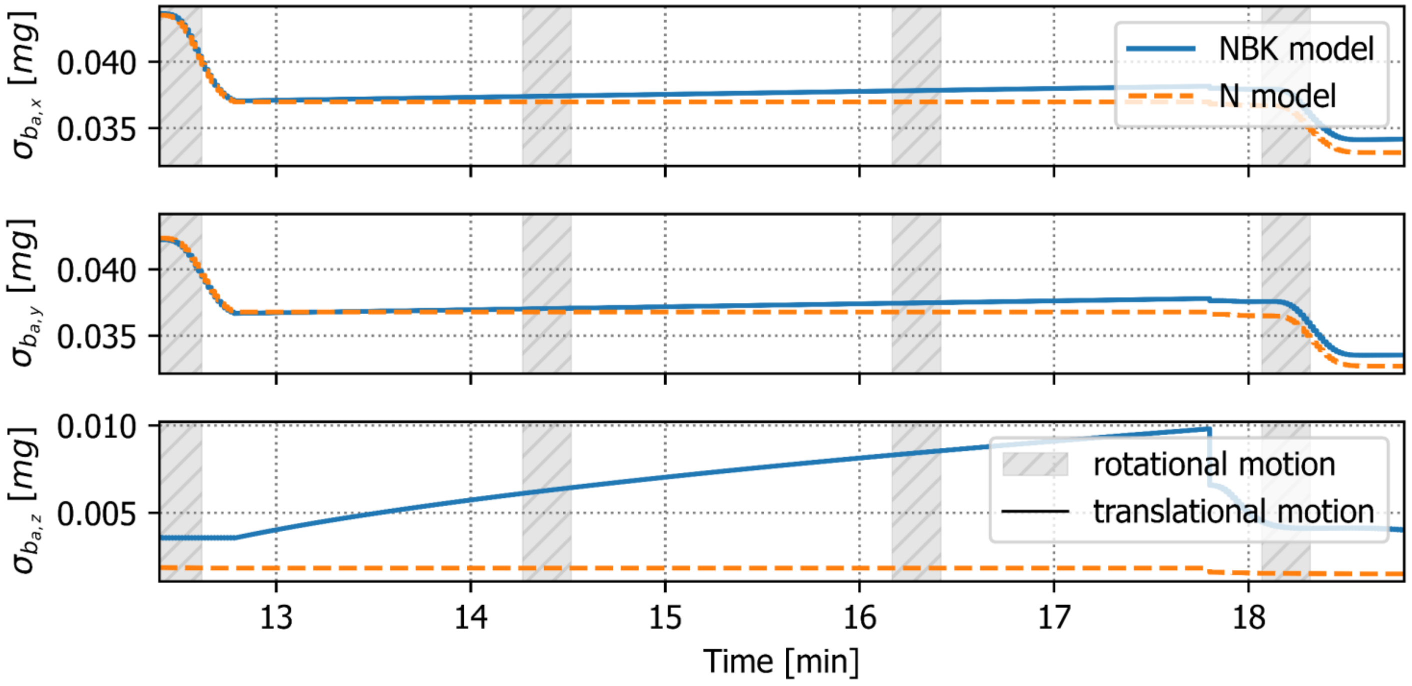

White noise of NBK model: $8.2\ \mu g/\sqrt{\mathrm{Hz}}$
White noise of N model: $50\ \mu g/\sqrt{\mathrm{Hz}}$

# Simulation results – Case 2 (GNSS outages for 5 min)



# Simulation results – Case 2 (GNSS outages for 5 min)



# Conclusions

- Successful identification and quantification of the different noise processes for the investigated IMU
- Incorporation of a detailed accelerometer bias model into the loose INS/GNSS integration architecture
  - Additional research to include quantization noise
- The largest contribution to the accuracy of the navigation solution came from errors in the gyros and not from errors in the accelerometers

# Outlook

- Solely estimated standard deviations were investigated
  - Investigation of true errors is possible in case of simulation studies
- Environmental induced errors are not taken into account by the AV method, but are frequently encountered in practice
  - Vibrations, temperature changes
- The conducted investigations will be verified on real world applications
  - Coverage of a wide range of vehicle dynamics

# References

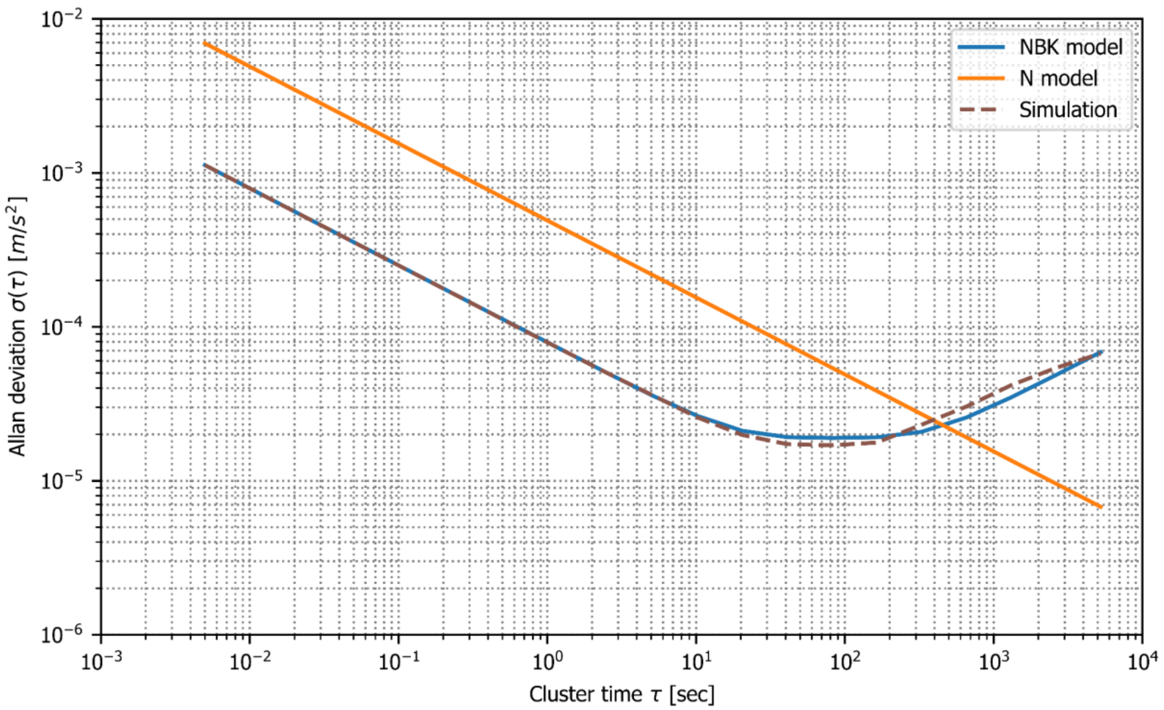

[1] D. W. Allan, Statistics of atomic frequency standards, Proc. of the IEEE, Volume 54, Issue 2, February, 1966, pp. 221-230..

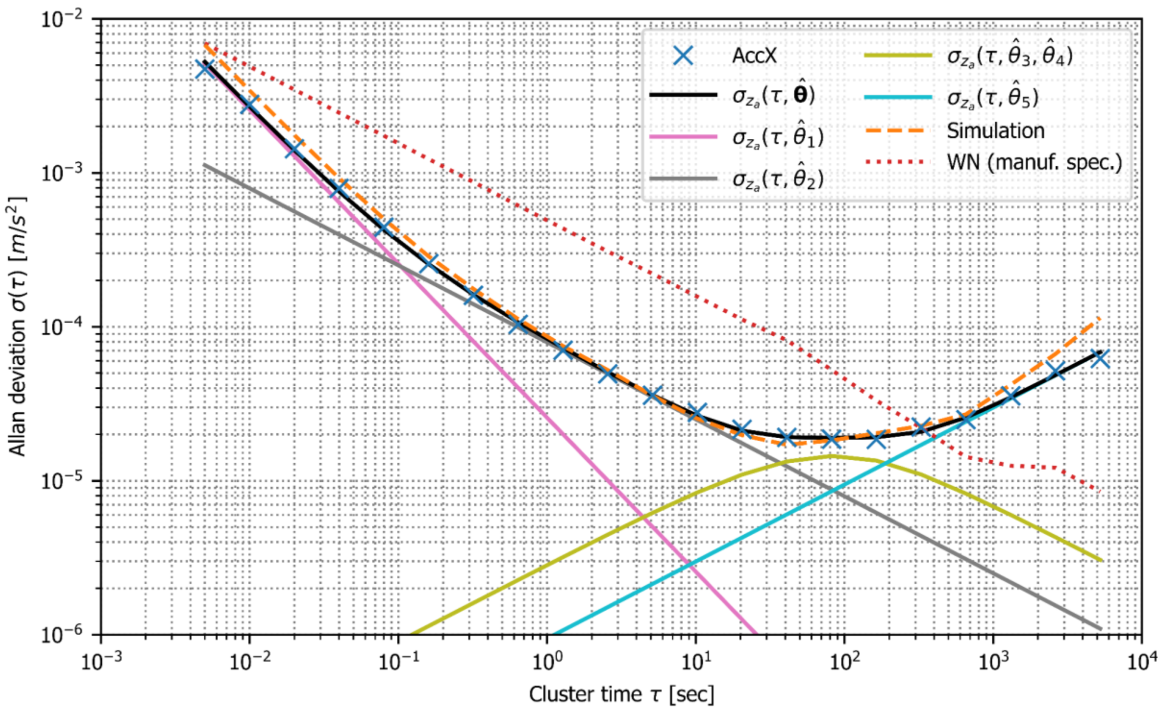
[2] StackExcahnge, How to interpret Allan Deviation plot for gyroscope? https://dsp.stackexchange.com/questions/53970/how-to-interpret-allan-deviation-plot-for-gyroscope/53993#53993.

[3] IEEE Standard 952: Specification Format Guide and Test Procedure for Single-Axis Interferometric Fiber Optic Gyros, IEEE, Tech. Rep. (1998), DOI: 10.1109/IEEESTD.1998.86153.

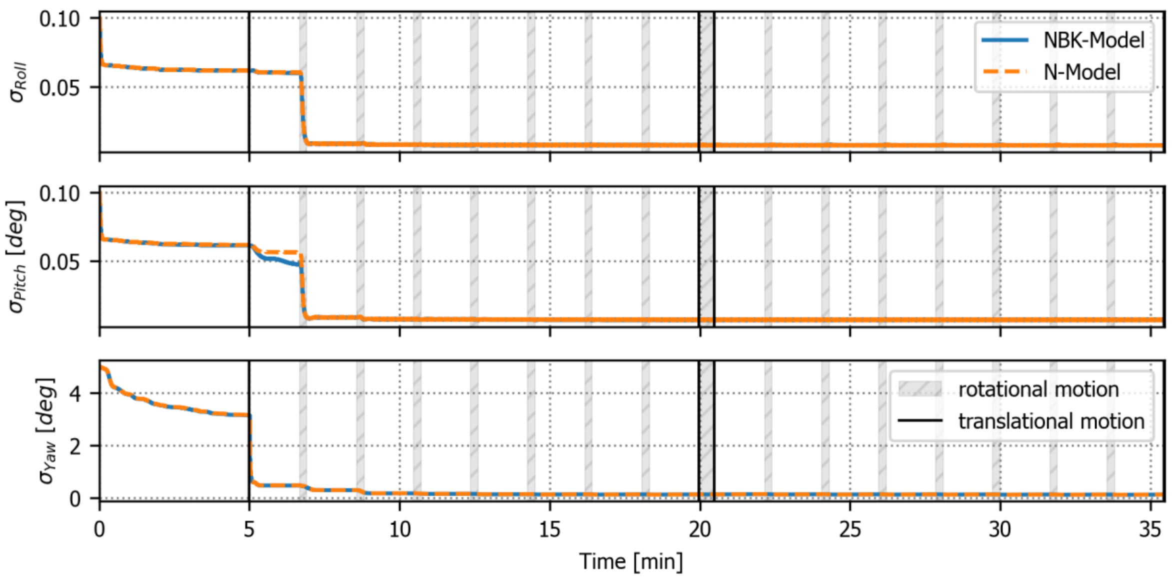
[4] P. Groves, Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems, 2nd ed. Artech House, 2013

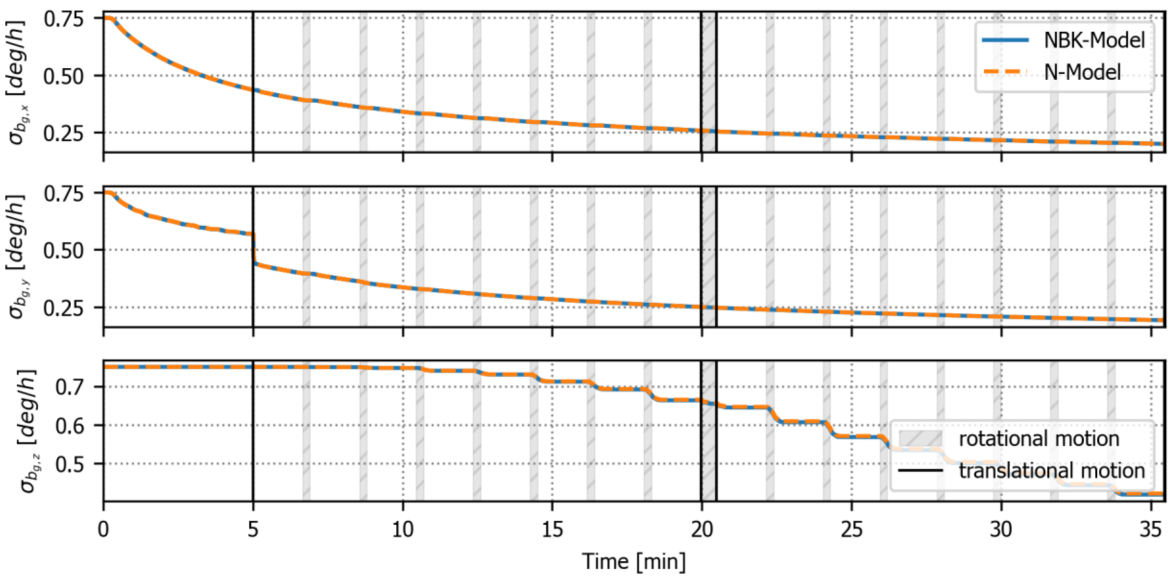
# Appendix I


NBK model
N model
Simulation
Allan deviation $\sigma(\tau)$ $[m/s^2]$
Cluster time $\tau$ [sec]

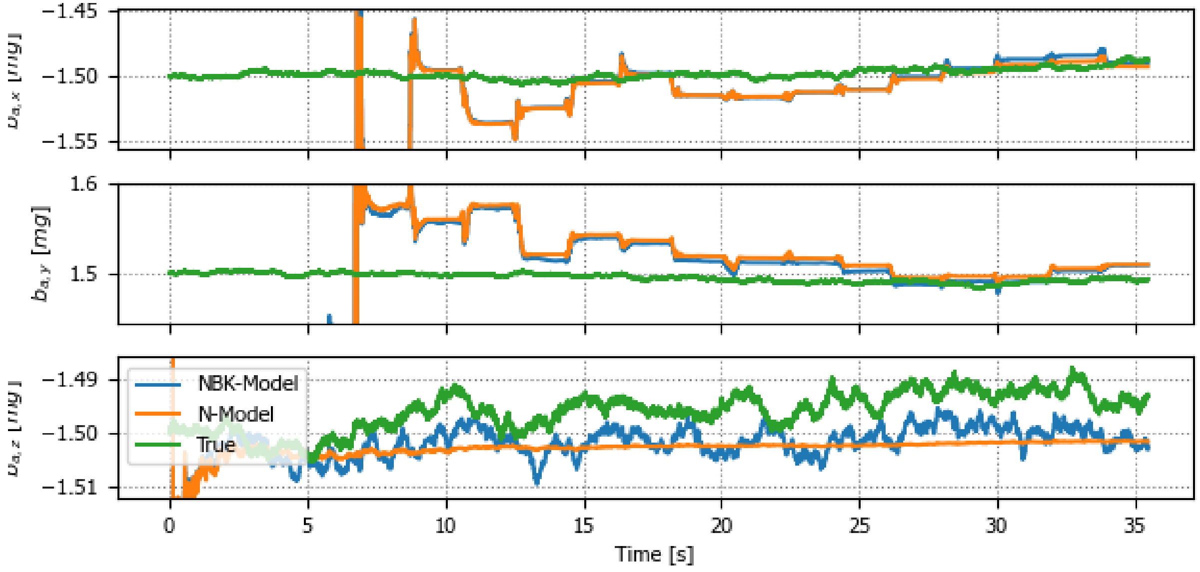

AccX
$\sigma_{z_a}(\tau, \hat{\boldsymbol{\theta}})$
$\sigma_{z_a}(\tau, \hat{\theta}_1)$
$\sigma_{z_a}(\tau, \hat{\theta}_2)$
$\sigma_{z_a}(\tau, \hat{\theta}_3, \hat{\theta}_4)$
$\sigma_{z_a}(\tau, \hat{\theta}_5)$
Simulation
WN (manuf. spec.)
Allan deviation $\sigma(\tau)$ $[m/s^2]$
Cluster time $\tau$ [sec]

# Appendix II.a

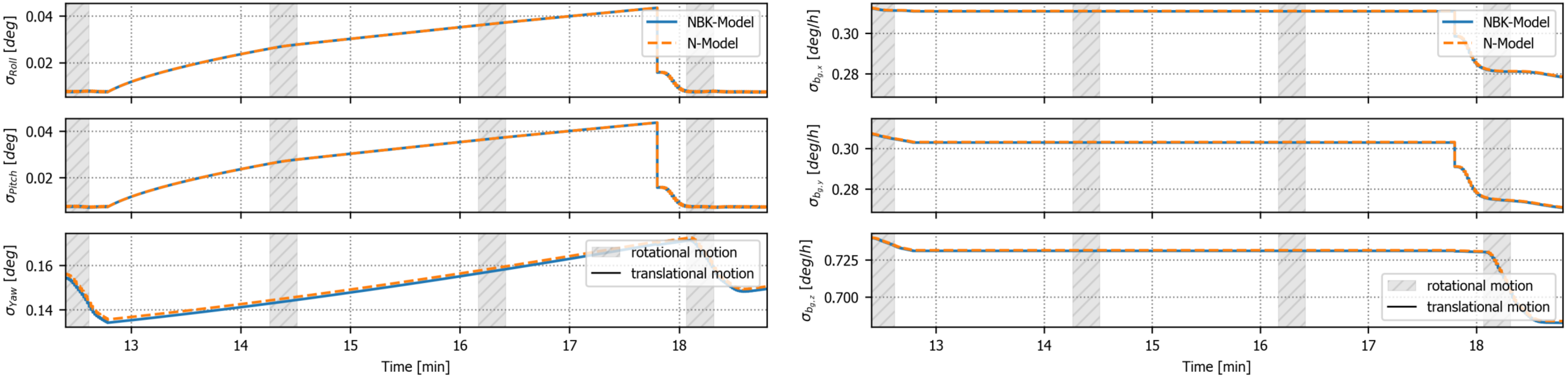


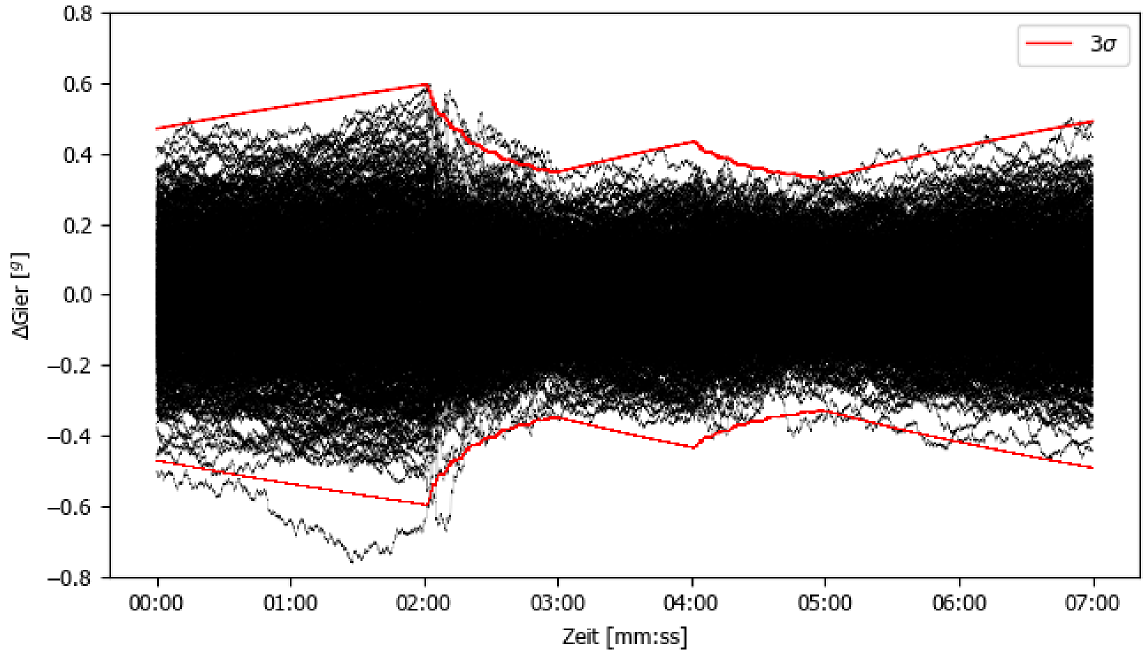
# Appendix II.b



# Appendix III



# Appendix IV


3σ
ΔGier [g]
Zeit [mm:ss]
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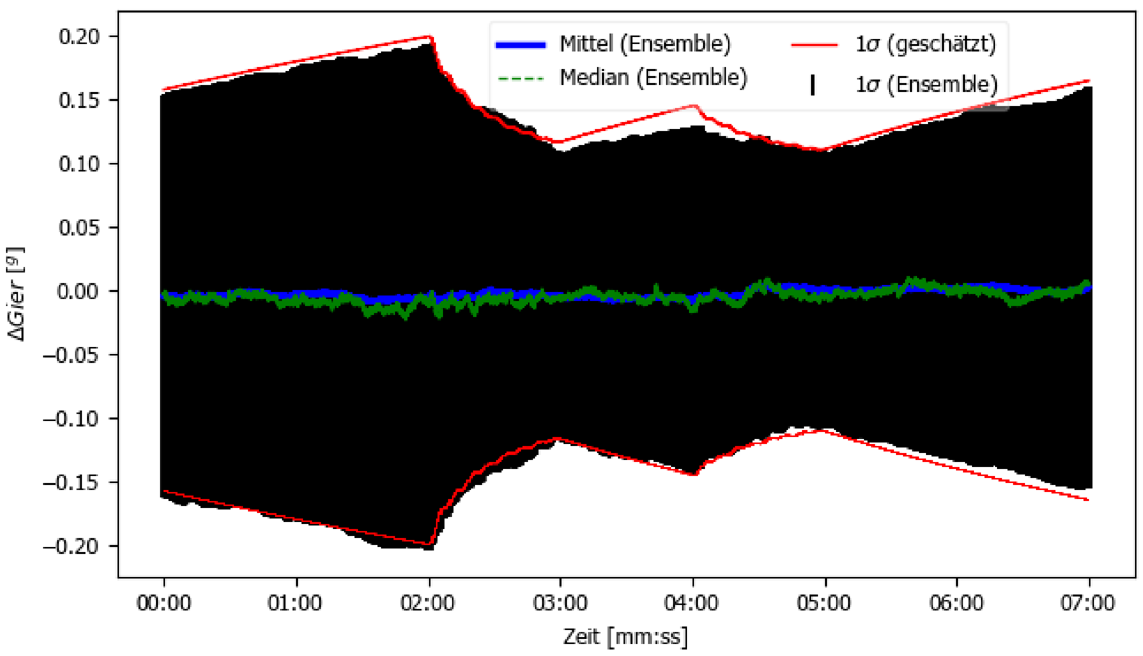
01:00
02:00
03:00
04:00
05:00
06:00
07:00
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8


Mittel (Ensemble)
Median (Ensemble)
1σ (geschätzt)
1σ (Ensemble)
ΔGier [g]
Zeit [mm:ss]
00:00
01:00
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05:00
06:00
07:00
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20

# Appendix V

**Tab. 1:** Estimated noise parameters of the x-axis accelerometer and the x-axis gyro

| Noise term | Noise Parameter | AccX | Manuf. Spec. | GyrX | Manuf. Spec. |
|---|---|---|---|---|---|
| **Quantization noise** | $S_A\ [m^2s^{-2}]$ | $1.4841e^{-5}$ | - | - | - |
| | $A\ [ms^{-1}]$ | | - | - | - |
| **White noise** | $S_N\ [m^2s^{-3}]\ ,\ [rad^2s^{-1}]$ | $6.2865e^{-9}$ | - | $5.3079e^{-10}$ | - |
| | $N\ [\mu g/\sqrt{Hz}]\ ,\ [deg/\sqrt{h}]$ | **8.1** | $< 50$ | **0.08** | $< 0.15$ |
| **Flicker noise** | $S_B\ [m^2s^{-5}]$ | $2.4360e^{-11}$ | - | - | - |
| | $B\ [\mu g]\ ,\ [deg/h]$ | **2.2** | $< 10$ | - | $< 0.1$ |
| | $T_B\ [s]$ | **45** | - | - | - |
| **Random walk** | $S_K\ [m^2s^{-5}]$ | $2.6731e^{-12}$ | - | - | - |
| | $K\ [ms^{-5/2}]$ | $1.6350e^{-6}$ | - | - | - |

# Appendix VI

- Overall PSD of the stochastic accelerometer errors (Superposition principle)

$$S_{z_a}(f) = S_{z_{a,A}}(f) + S_{z_{a,N}}(f) + S_{z_{a,B}}(f) + S_{z_{a,K}}(f)$$

- AV is related to the PSD via

$$\sigma_z^2(\tau) = 4\int_0^\infty S_z(f)\frac{\sin^4(\pi f\tau)}{(\pi f\tau)^2}df$$

- Overall AV for the accelerometers

$$\sigma_{z_a}^2(\tau) = \sigma_{z_{a,A}}^2(\tau) + \sigma_{z_{a,N}}^2(\tau) + \sigma_{z_{a,B}}^2(\tau) + \sigma_{z_{a,K}}^2(\tau)$$

$$= \frac{3S_{a,A}}{\tau^2} + \frac{S_{a,N}}{\tau} + \frac{S_{a,B}T_B^2}{\tau}\left[1 - \frac{T_B}{2\tau}\left(3 - 4e^{-\frac{\tau}{T_B}} + 4e^{-\frac{2\tau}{T_B}}\right)\right] + \frac{S_{a,K}}{3}\tau$$

- Defining the parameter vector $\boldsymbol{\vartheta}$

$$\boldsymbol{\vartheta} = [S_{a,A} \quad S_{a,N} \quad S_{a,A} \quad T_B \quad S_{a,K}]$$