25 Years of Teaching Least Squares Adjustment

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SUMMARY

In geodesy the network adjustment is one of the basic tasks in data processing. The results of least squares adjustment, coordinates and heights of the points, are the main products that the geodetic surveyors provide to the society. It may be hard to use the commercial software or understand the results if the knowledge of the adjustment calculus is missing. The software packages are often like black boxes which have some input and after pushing the button some output.

The author has been happy to give lectures and exercises in adjustment calculus over 25 years in Helsinki University of Technology and Aalto University. During the teaching period from the end of the 1980s until now the computers and software used in exercises have been improved. The working environment in classrooms has changed. Paper, pen and pocket calculators have been replaced with efficient computers with mathematical software like MATLAB® (The MathWorks, Inc.), GNU Octave (Free software, Copywright John W. Eaton and others), Maple (Waterloo Maple, MapleSoft) and Wolfram Mathematica.

An overview of the experiences and developments in teaching will be presented in this paper. The learning of adjustment calculus is presented with Bloom's taxonomy.

The importance of teaching least squares technique has not come to its end. With the other estimation techniques it is still the basic tool in geodetic computation. The new course, "Least Squares Methods in Geoscience", in the teaching program of Aalto University will start in 2017.

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1. INTRODUCTION

The Least squares method has been known since French mathematician, Adrien-Marie Legendre published it in 1806. Also Gauss discovered the method, even earlier than Legendre, but he did not publish it. The method was soon adopted in geodesy and it is still one of the most used methods in geodetic computation.

To get reliable coordinates or heights we need geodetic networks (GPS networks, terrestrial networks, levelling networks). The end products of network measuring process are coordinates or heights with estimated uncertainties. They do not come as granted. We should be professional proud of the result. Some other uses our product as output data.

The commercial geodetic or surveying software use the least squares method frequently, but users may not be aware about that. In order to be able to interpret the reports of computations or supervise the quality of networks one needs to know at least the basics of the theory of least squares. The adjustment calculus is one of the corner stones of the professional knowledge of geodetic surveyors.

Teaching least squares method has long traditions. I am proud to be one teacher in the long chain. I started in 1984 as an assistant in the Department of Geodesy and Cartography in Helsinki University of Technology. After programming some network adjustment programs I was asked to give exercises in the course of adjustment calculus. And when the professor of Geodesy retired in 1990, I was asked to be the responsible teacher of the course. It is not easy task to teach least squares adjustment. The subject has been considered to be difficult. It is not popular course.

The point of view of the teaching can be the contents of the course, activities, material and environment of the teaching or goal of the learning. Pedagogical methods have changed from monologue of the teacher to the direction of the inquiry-based or problem-based learning. In this paper I survey the development that I have recognized during last 25 years. The goals of the learning are still the same: students should be able to apply least squares method and analyze the results they get.

In the section two I present the goals of the learning in the least squares adjustment with Bloom's taxonomy. In the section three I present the content of the course. The section four concerns the changes in learning environment, tools and material.

2. GOALS OF LEARNING

The goals of teaching and the goals of learning should be aligned. The goals depend on the students, what is the output level or background of them.

2.1 Bloom's taxonomy

Bloom's taxonomy is widely used and cited in educational literature. There are hundreds of web pages available with nice figures of the taxonomy: pyramids, wheels, ladders, steps etc. Benjamin Bloom, Max Englehart, Edward Furst, Walter Hill, and David Krathwohl published the *Taxonomy of Educational Objectives* in 1956. They categorize the cognitive levels or goals of learning. The six original categories are**: I Knowledge, II Comprehension, III Application, IV Analysis, V Synthesis, and VI Evaluation**. [1],[2]

The Bloom's taxonomy is nice tool for planning the course. I apply the taxonomy in Figure 1 to the learning of least squares adjustment. I use the original levels while there also is the revised Bloom's taxonomy with slightly different names for the levels. For each level I present some activities which can help to reach the goals.

Bloom's taxonomy can be used in assessment of learning too.

2.2 Learning activities and exercises for students in Bloom's taxonomy

There is no guarantee that learning process of human being follows these steps but when we are teaching a tool, in this case the least squares adjustment method, it is reasonable to use step by step approach.

The lowest level **(Knowledge)** could be the goal of the first lecture. The methods required to reach the goal are traditional lecture and some activities during lecture. The activities can be just discussion or interview of students, whether they know or have used least squares method.

In the second level **(Comprehension)** the general solution of least squares is derived with students. The examples of typical cases of observation equation model, condition equation model or mixed model are introduced. Teaching method and material depend on the number of students. The possibility to use some commercial adjustment software may help. The variance propagation law is derived with examples.

In order to reach the learning goals of the third level **(Application)** students apply the least squares method with MATLAB® or GNU Octave to the levelling network and some nonlinear case as trilateration. They are guided to form observation equations, normal equations and solve for the parameters. They are guided to calculate the reliability measures and basic tests. The assignment is related to some adjustment problem. I have used trilateration, 3D-local network adjustment, 3D Helmert transformation and sphere fitting. Students program the solution and analysis with MATLAB®. The other assignment is related to the variance propagation law. My favorite

assignment has been the precision of the results of javelin. It includes the listing the uncertainty sources of the measurement. It also makes student to think the difference of precision and accuracy.

In the fourth level **(Analysis)** students get some real case to study. Using the tools from previous levels they analyses the measurement plan and make decisions and suggestions for improvements. They practice the data snooping with real data. They either make the measurement plan or analyze the adjustment report. They must find if there is outliers and they must calculate some reliability measures and make decisions.

2.3 Levels V and VI in advanced studies

The level III Application should be achievable in basic course. The level IV is possible achievable in basic course but the next levels need time and more experience. They are expert levels

The level five **(Synthesis)** will be reached in the advanced course. It does not belong to the basics. The advanced students (at least master level) can do this

The level sixth **(Evaluation)** is the task for graduated students or professionals in the research work.

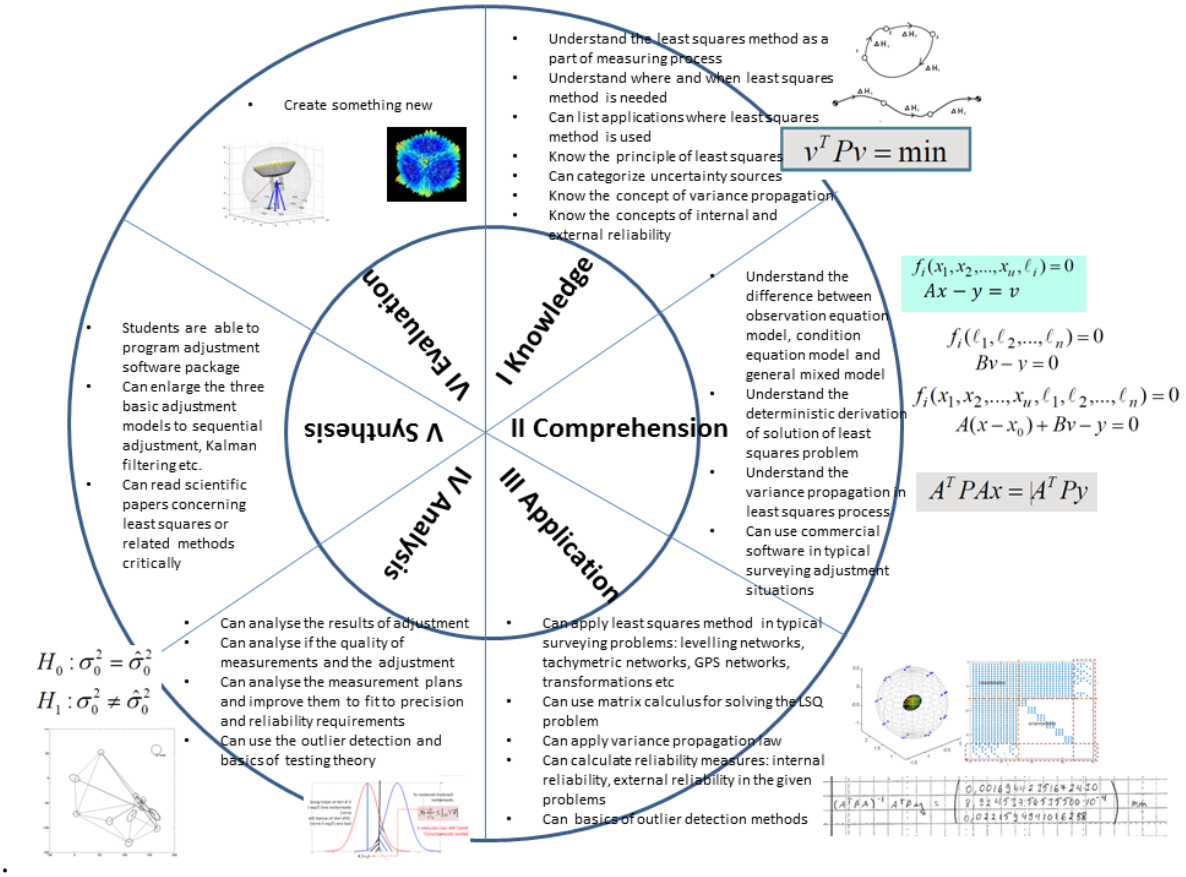

Figure 1. Bloom's taxonomy wheel in the learning of least squares adjustment

3. PLANNING THE CONTENT

The requirements to the content of the course should come from learning goals. When the goals are clear it is quite easy to determine the content. It may follow some textbook if it is alignment with goals.

The scientific point of view to the content of the course in the case of least squares method is the theory of least squares: derivation of the general solution of the least squares problem in the deterministic way with matrix calculus, showing the relationship of the least squares adjustment and the Best linear Unbiased Estimation and Maximum Likelihood estimation, showing the geometric interpretation of least squares solution. Also the testing theory and uncertainty analysis belong to the course. The questions of linear algebra and numerical analysis should be included to the content. In the professional point of view the content is more applications: how the least squares method is applied in network adjustments, transformations and fitting problems. In Figure 2 we show the content of the basic course of adjustment calculus

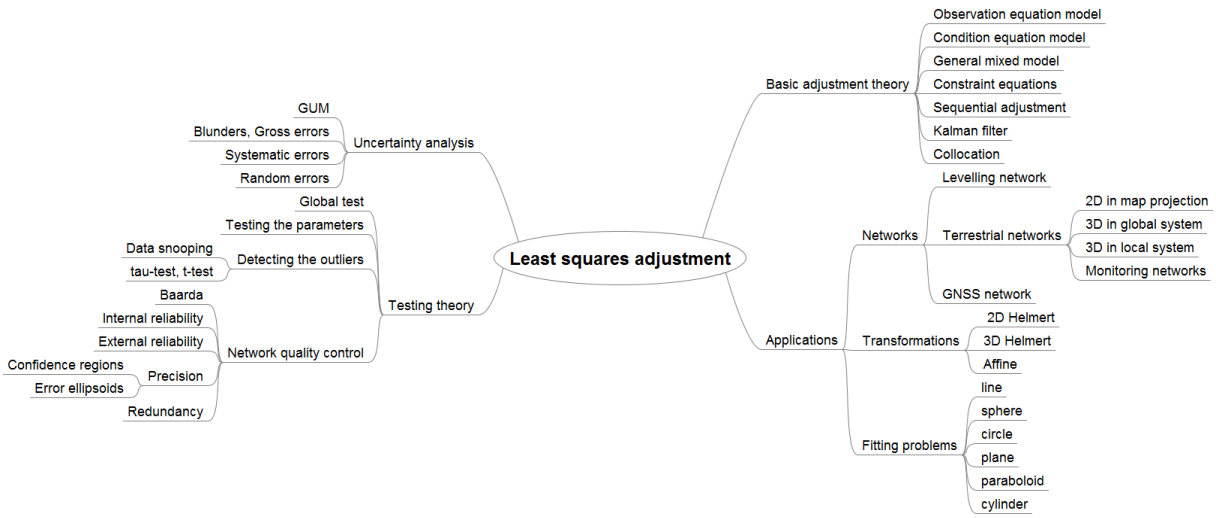

Figure 2 Content of the course of adjustment calculus

4. LEARNING ENVIRONMENTS

The classroom, timetables and the given resources form the framework of the teaching and learning. Sometimes they are restrictive and even prevent the learning if they are not planned together with the learning goals.

4.1 Classroom, from auditorium to computer classes

The auditorium is good place to teach if I like to present a monologue. I can dump a lot of details or give a brilliant speech for the student without knowing how they learn. In auditorium it is not easy to get response from students.
Sometimes it is still necessary to just give lectures. Some activities during the lecture help keeping the focus on the subject and guide from teacher centered to student centered learning. Instead of

deriving the formulas on slides or on blackboard, I can let students to do it as a group work. For example they can form observation equations or linearize equations. Mixing the traditional classroom exercises and lectures activate students to think.

The classroom exercises are replaced with home exercises or guided computer class exercises. In guided exercises the students perform the given tasks individually or in small groups while the teacher walks around giving help when needed.

4.2 Lecture material, textbooks, exercises and assignments

The role of lectures has changed. Today they should be more like guiding to the information sources and guiding to ask the relevant questions. Nowadays it is not necessary to give all the details in slides or on the blackboard, because everyone has the access to the necessary information easily. Good textbooks and internet material are available.

In 1990 we still have paper and pen exercises in classrooms. The examples were easy, because they were solved for with pocket calculators during the class. The modern mathematical software like MATLAB®, GNU Octave, Waterloo Maple, Wolfram Mathematica give us possibilities to solve for more difficult adjustment problems in classroom exercises. We can have more data in examples. For example in order to learn outlier detection we can use the data snooping method with real observation data with thousands of observations which was not possible in pocket calculator era. Also illustrating the results, residuals, error ellipsoids, network structure or matrix sparse structure is easy with mathematical software. We started MATLAB-exercises in 2004. The exercises have varied from independent work to the guided case studies of network adjustment.

The Figure 3 and Figure 4 are examples of student works in 1991 and in 2014 respectively. In 1991 we have possibility to program with FORTRAN or C and in our PC class there was mathematical software Derive (Chartwell-Yorke Ltd.) installed in microcomputers. Also the students were provided the matrix inverse program, which read the matrix from ASCII file and wrote the inverse to the other file if the matrix was not singular. Most of the students still chose the pocket calculators, which had the matrix inverse property or combination of pocket calculators and matrix inverse program. The report was often written by hand. Nowadays we have computer classes with powerful mathematical software. The students can have student version of MATLAB in their own laptops or remote access to the University computers. It makes the learning more comfortable. The reports are not printed but submitted into the study portal.

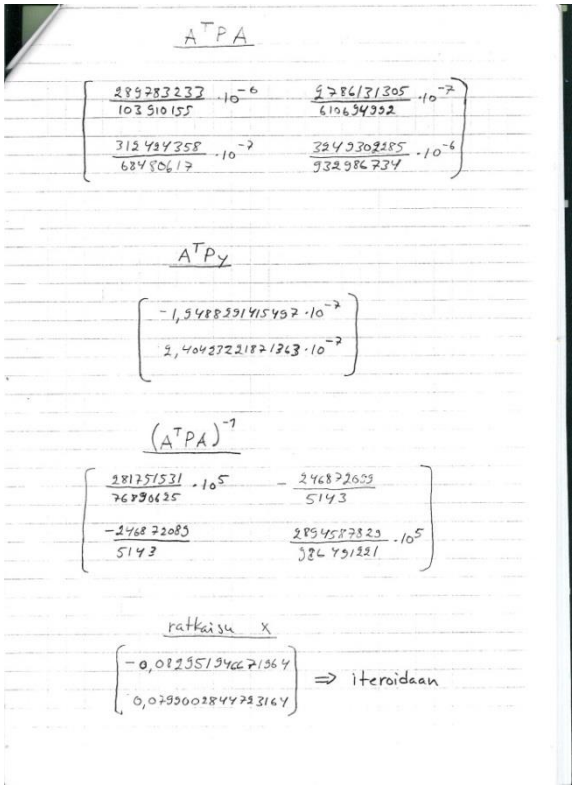

Figure 3 An example of the student work: " the adjustment of trilateration" calculated with pocket calculator and reported with paper and pen in 1990. Document was about 15 pages.

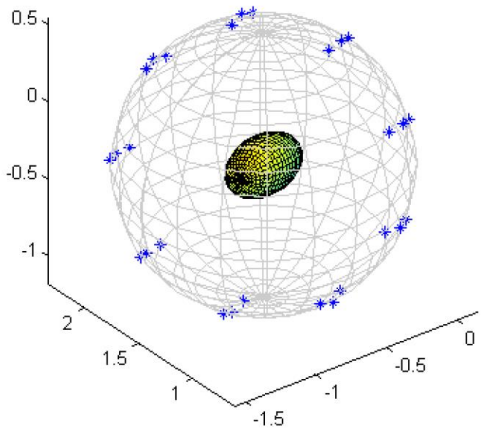

Figure 4 An example of the student work: "fitting the sphere" programmed with MATLAB in 2014

The coarse subjective timeline of developements in teaching the least squares adjustment is sketched in Figure 5. The influence of the developements in web-based services and learning environments on the teaching is clear. The developments in exercises and assignments around the year 2005 are related to my educational studies. Helsinki University of Techology (Aalto university since 2010) offered quite large university pedagogical training, YOOP for teachers in 1999-2009.

Figure 5 Timeline of developments in the teaching of least squares adjustment. Gray color is related to the management of studies, orange to the delivering the lecture material, pink to the tools, yellow to the teaching experiments and blue for reading material.

5. FUTURE

The new course "Least Squares Method in Geosciences" started in January 2017 [3]. The new course is quite challenging and large. The first round of lectures and assignments are now behind. We had at first basics and then geodetic, photogrammetric, collocation and Kalman filter applications. The assignments were quite demanding. We need critical self-assessment and courage to do necessary improvements after we get student feedback. The time of writing the students have deadlines of the assignments.

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BIOGRAPHICAL NOTES

The author works as a Senior Research Scientist in Finnish Geospatial Research Institute. She is responsible of the local tie measurements and analysis in Metsähovi. She has given lectures and exercises in Helsinki Technical University and Aalto University since 1984.

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