The Comparison Of The Adjustment Methods In Geoid Determination Method

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Key words: The Least Square Method (LS), The Least Absolute Value Method (LAV), Orthometric and Ellipsoidal Height, Multiquadratic Interpolation, Polynomial Interpolation

SUMMARY

The measurement number is bigger than required measurement in geodetic application generally. In this case the adjustment methods are applied for unique solution. The most used adjustment are the least square method (LS) and the least absolute value method (LAV).

Geoid determination is an application used frequently in geodesy. In geodetic relations between orthometric height and ellipsoidal height obtained from geoid determination. Orthometric heights are used in engineering applications but the measurements of orthometric height are quite difficult. The ellipsoid height is obtained from the space geodesy technique. A lot of methods are used for the determination of geoid. The methods of this study; are polynomial and multiquadratic interpolation. A great number of studies in which polynomial geoid determination technique is used have been conducted in our country. Some of these studies have formed numerical height models by using polynomial interpolation technique. The multiquadratic interpolation of the purpose is to define the research are with only one function.

In this study, the adjustment methods of geoid determination methods were introduced theoretically. The geoid determination was realized using the polynomial and multiquadratic methods according to LS and LAV method. Thus it decided to the best method for geoid determination.

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1. INTRODUCTION

The geoid determination is the most important problem for scientist interested in the earth. There are a lot of areas interested in geoid like geodesy, geophysics, geography etc. (Akçın, 2001). The geoid called the surface closed the average sea surface and formed by the combination of the points have got zero potential value. The geoid is a complex surface and it is not easy defined as mathematically. In the geodesy the measurements on the physical earth, but the calculation of measurements is done on the reference surface. Thus, the difference between the reference ellipsoid was called geoid undulation. The geoid determination methods had been developed to obtained geoid undulation values (Bolat ,2011).

The vertical datum is formed by geoid, that is, average sea level. The point height on earth is found as dependent on the average sea level formed for the area to be mapped. The current vertical datum used in the maps in our country was determined with the average of measurements done between the years 1936-1970 in the sea level measurement (tide gauge) station built in Antalya (Düzgün, 2010).

The point height on earth is determined distance between datum and local surface. . Height, which is the coordinate parameter, is measured according to two different surfaces. The distance along the plumb line of the geoid is called the orthometric height, while the distance of reference ellipsoid along the normal of ellipsoid is called ellipsoidal height.

The ellipsoidal heights can be obtained using the Global Positioning System easily. But the orthometric height is used in practical applications because of leveling surface and plumb line. Thus, relationship should be determined between ellipsoidal heights and orthometric heights.

The relationship of ellipsoidal height (h) and orthometric height (H) given in Figure 1. can be established using N is geoid undulation (Kartal, 2001).

$$N = h - H \tag{1}$$

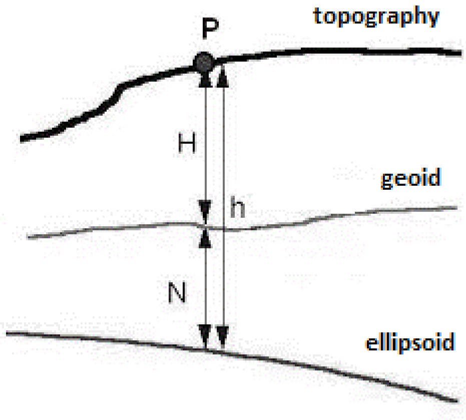

Figure 1. The height used in Geodesy

The 42nd item of Large Scale Map and Map Data Production Guide mentions geoid undulation. There are a lot of methods for the determination of geoid undulation (BÖHHBÜY, 2005, Kırıcı U., 2016). The polynomial and multiquadratic interpolation methods had been used in this study.

2. INTERPOLATION METHODS

2.1. Polynomial Interpolation

The polynomial geoid determination technique is based on the determination of polynomial surface. This method is mostly common because of understandability and easy solvability. There are a lot of application in our country. Some of these applications is realized the surface determination, the others are investigated the sensitivity (Bolat , 2011). The surface used while determining geoid is generally expressed in high degree polynomials with two variables. The orthogonal polynomials can be represented are as follows;

$$N_{(x,y)} = \sum_{i=0}^{m} \sum_{\substack{j=k-i \\ i=0}}^{k} a_{ij} x^i y^j \quad \textbf{(2)}$$

Here a_{ij} shows polynomial coefficients, m shows the degree of polynomial and (x, y) shows the plane coordinates of the points. The polynomial degree should be chosen and the polynomial equation should be formed for this degree. Polynomial equation can be written for 3rd order polynomial are as follows;

$$N = a_{00} + a_{10}X + a_{01}Y + a_{20}X^2 + a_{11}XY + a_{02}Y^2 + a_{30}X^3 + a_{21}X^2Y + a_{12}XY^2 + a_{03}Y^3 \quad \textbf{(3)}$$

Generally in this problem the point coordinates are taken as measurements. The measurements can be include some outliers. In this case it is prefer that the number of measurements n are selected bigger than numbers of unknowns u and the adjustment solution is realized for determining the coefficients. The solution of this problem is realized according to the adjustment method and the unknown coefficients are obtained. Then, the geoid undulation of new point can be calculated using obtained polynomial (Kırıcı and Şişman, 2014).

2.2.Multi-Quadratic Interpolation

The multiquadratic interpolation was first used by Hardy in the representation of disorderly measured topographic surfaces. In this method, the purpose is to define the research using only one function. The first stage of the multiquadratic interpolation method is the calculation of ΔN_i of reference points. the second stage is the calculation of the unknown coefficients of the polynomial according to adjustment method. ΔN_i is obtained as follows;

$$\Delta N_i = N_i - N(x_i, y_i) \tag{4}$$

The residual value of the undulation in (x_0, y_0) interpolation point is as follows;

$$\Delta N_0 = N_0 - N(x_0, y_0) \tag{5}$$

$$\Delta N_0 = \sum_{i=1}^{n} c_i \theta(x_0, y_0; x_i, y_i)$$

$$\theta(x_0, y_0; x_i, y_i) = \left[(x_i - x_0)^2 + (y_i - y_0)^2\right]^{1/2} \tag{6}$$

$$c = A^{-1}.\Delta N \tag{7}$$

N_0 undulation value calculated by the equation;

$$N_0 = N(x_0, y_0) + \sum_{i=1}^{n} c_i \left[(x_i - x)^2 + (y_i - y)^2\right]^{1/2} \tag{8}$$

The advantages of multiquadratic method are: (Teke and Yalçınkaya,2005).

- Even if the reference points are not homogenously distributed, the results of surface modeling are barely affected.
- In case of an increase in the distance from reference points to the calculated point, the contribution to surface modeling decreases as much as the increase.
- There aren't any overlay remains for behind the reference points. (Yaprak,2007).

3. ADJUSTMENT PROCEDURE

In applied sciences, measurements are made more than the number of unknowns in order to increase the accuracy and precision obtained from measurements and the results of measurements. In a problem, the number of unknowns parameters is equal to the number of sufficient calculation and it is shown with u. If the number of measurements (n) is higher than the number of unknowns u, there is the more solution of the problem. In such as system, adjustment is made to obtain the only significant result. The objective of adjustment is to find out the most suitable and highest probability value of the unknown and unknown functions without leaving out any measurement from measurement groups which do not contain gross or systematic error (Wang, 1992).

In adjustment, solutions are made according to an objective function in order to determine unknown functions. In order to find out X unknown factors with balance calculation of the $\hat{\ell}$ measurement group, mathematical model which shows functional and stocostic relationship between unknowns.

$$\hat{\ell} = \Phi_i(x_1, x_2, \ldots\ldots, x_u) \qquad Q_{\ell\ell} = P^{-1}; \quad C_{\ell\ell} = \sigma_0^2 Q_{\ell\ell} \tag{9}$$

Gauss-Morkoff model, which is also known as linear mathematical model, is obtained by linearizing the equations above. (Wang, 1992, Vanicek, Wells, 1972).

$$E\{\hat{\ell}\} = \underline{\ell} + \underline{v} = \underline{A}x \qquad \underline{Q}_{\ell\ell} = \underline{P}^{-1}; \quad \underline{C}_{\ell\ell} = \sigma_0^2 \underline{Q}_{\ell\ell} \tag{10}$$

In equations (9) and (10), A is the design matrix of the mathematical model. Mathematical model given with (9) is solved according to a objective function. Objective function is chosen according to the minimum number of measurement residual. LS method solves with $\|Pvv\| = [Pvv] = \min.$ objective function, LAV method with $\left||pv|\right| = [P|v|] = \min.$ and LS method with $\left||P[v{:}\, v_{A_2}]P|\right| = \min.$ (Sisman, at al., 2013, Kırıcı and Sisman, 2015).

3.1.The Least Squares Method

The least squares method (LS) explained by Carl Friedrich Gauss in 1795 and Legendre in 1805. This method is used in many different applications (Sisman, 2014). Unknown parameters calculated with the following equation in this method.

$$\underline{X} = \left(\underline{A}^T \underline{Q}_{\ell\ell}^{-1} \underline{A}\right)^{-1} \underline{A}^T \underline{Q}_{\ell\ell}^{-1} \underline{\ell} \tag{11}$$

Root mean square error (RMSE);

$$m_0 = \pm\sqrt{\frac{\underline{V}^T \underline{P}\underline{V}}{f}} \quad ; f = n - u \tag{12}$$

The measurement errors of the LS method influence the residual of other calculations. Thus, this correction value may not always be due to an error in the measurement. This situation is called the spread and storage effect of LS method. Different solution methods can be conducted for the analysis of spread and storage method.

3.2. The Least Absolute Value Method

LAV method is a method given by Laplace in 1789 which is used to solve many different problems. Mathematical model given by (10) is solved according to objective function with the least absolute total method $||\text{Pv}|| = [P|v|] = \text{min}$. In this operation, direct solution is not possible except special cases. The solution can be found as trial and error or linear programming problem. LAV method includes unknown parameters such as $\underline{X}$ and $\underline{V}$.

New unknowns are as follows for linear programming;

$$X = X^{+} - X^{-}; \quad X^{+}, X^{-} \geq 0,$$
$$V = V^{+} - V^{-}; \quad V^{+}, V^{-} \geq 0 \tag{13}$$

$$\begin{bmatrix} A & -A & -I & I \end{bmatrix} \begin{bmatrix} X^{+} \\ X^{-} \\ V^{+} \\ V^{-} \end{bmatrix} = [\ell], \tag{14}$$

$$f = b^{T} X = [P|V|] = P^{T} V = p^{T} \lfloor V^{+} \quad V^{-} \rfloor = \text{min.}$$

In the prediction of parameters in LAV method, measurements made as much as the number of unknowns are used and these measurements are considered as unerroneous. Residuals values are calculated from the solution of measurements which are not used in the prediction of unknown parameters according to LAV method. Thus, the spread and reflection of the other measurements' errors to the residual of measurements disappear (Bektas, Sisman, 2010, Sisman, 2014, Kırıcı, Sisman, 2015).

4. THE NUMERICAL APPLICATION POLYNOMIAL AND MULTI-QUADRATIC INTERPOLATION

In this application, the coordinates of 20 points in Samsun region and their ellipsoidal and orthometric heights were used. The unknown parameters were found for these points according to both polynomial and multiquadratic method according to LS and LAV method. Comparisons were made between four methods.

The first order polynomial was used in the calculations.

$$N = a_{00} + a_{10} X + a_{01} Y \tag{15}$$

The values found are given in Table 1. These operations were made in Matlab programming language.

Table 1. The finding unknown parameters due to four different methods

X	Polynomial		Multi-quadratic	
	LS	LAV	LS	LAV
a_{00}	28.17204963	28.16889046	226.17233035	218.06967963
a_{10}	-0.03924029	-0.03776712	-0.00003924	-0.00003777
a_{01}	-0.03436085	-0.03185382	-0.00003436	-0.00003185

The equation of polynomial and multiquadratic interpolation are given follow;

$$N = 28.1720 - 0.0392\,X - 0.0344Y$$ Polynomial Interpolation (LS)

$$N = 28.1689 - 0.0378\,X - 0.0319Y$$ Polynomial Interpolation (LAV)

$$N = 226.1723 - 0.000039\,X - 0.0000344\,Y$$ Multiquadratic Interpolation (LS)

$$N = 218.0697 - 0.0000378\,X - 0.0000319\,Y$$ Multiquadratic Interpolation (LAV)

5. RESULTS AND DISCUSSION

Geoid undulations are calculated for 10 points application of the selected using the equ (1),(2). For testing these method. The obtained geoid undulation are given in Table 2.

Table 2. Selected points in the study area

Nokta Adı	East	North	Orthometric Height(H)	Ellipsoidal Height(h)	N=h-H
409	542850.957	4567840.372	1.895	30.236	28.341
413	542336.182	4569055.295	2.593	30.798	28.205
466	546205.568	4564605.175	8.893	37.141	28.248
473	548671.970	4563911.807	10.183	38.296	28.113
472	548221.835	4563915.699	12.231	40.362	28.131
408	542777.297	4567328.650	2.188	30.521	28.333

471	547780.410	4563813.093	16.606	44.748	28.142
469	547051.761	4563884.710	13.185	41.368	28.183
399	545088.541	4564666.379	10.689	39.010	28.321
291	541756.883	4568794.285	1.314	29.639	28.325

Geoid undulation values were compared to obtain from formula (1) and (2). Then Table 3 show found residuals (V) according to Polynomial interpolation and Table 4 show fond residuals (V) according to Multiquadratic interpolation.

Table 3. The correction value according to Polynomial Interpolation

Point Number	Geoid Undulation (N_m) (m)	Calculated Geoid Undulation(N_c) (m) (LS)	V $(N_m - N_c)$ (m)	Calculated Geoid Undulation(N_c) (m) (LAV)	V $(N_m - N_c)$ (m)
409	28.341	28.175	0.166	28.168	0.173
413	28.205	28.145	0.060	28.139	0.066
466	28.248	28.186	0.062	28.184	0.064
473	28.113	28.129	-0.016	28.131	-0.018
472	28.131	28.144	-0.013	28.145	-0.014
408	28.333	28.197	0.136	28.190	0.143
471	28.142	28.163	-0.021	28.163	-0.021
469	28.183	28.185	-0.002	28.184	-0.001
399	28.321	28.222	0.099	28.217	0.104
291	28.325	28.175	0.150	28.167	0.158

Table 4. The residuals according to Multiquadratic Interpolation

Point Number	Geoid Undulation	Calculated Geoid Undulation(N_c) (m)	V $(N_m - N_c)$ (m)	Calculated Geoid Undulation(N_c) (m)	V $(N_m - N_c)$ (m)

	(N_m) (m)	(LS)		(LAV)	
409	28.341	28.277	0.064	28.265	0.076
413	28.205	28.247	-0.042	28.235	-0.030
466	28.248	28.289	-0.041	28.280	-0.032
473	28.113	28.232	-0.119	28.228	-0.115
472	28.131	28.247	-0.116	28.242	-0.111
408	28.333	28.300	0.033	28.286	0.047
471	28.142	28.266	-0.124	28.260	-0.118
469	28.183	28.288	-0.105	28.280	-0.097
399	28.321	28.325	-0.004	28.313	0.008
291	28.325	28.278	0.047	28.263	0.062

When the measurement value of the approach test Tables 3 and 4 was observed Multiquadratic interpolation gives beter results than polynomial interpolation.

In this study conducted for geoid determination for Samsun region. The polynomial interpolation and multiquadratic interpolation according to LS and LAV method were compared first by using the ellipsoidal and orthometric heights of 20 points. After the unknowns parameters in first degree polynomial were found according to both methods. For testing these method another 10 points were used in the research area and residual were found. It was found that the multiquadratic interpolation method gave more accurated results.

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BIOGRAPHICAL NOTES

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