Transformation pipelines for PROJ.4

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SUMMARY

For more than 2 decades, PROJ.4 has been the globally leading map projection library for open source geospatial software. While focusing on mathematically well-defined 2D projections from geographical to planar coordinates, PROJ.4 has nevertheless, since its introduction in the 1980s, provided limited support for more general geodetic datum transformations, and has gradually introduced a higher degree of support for 3D coordinate data and reference systems.

The support has, however, been implemented over a long period of time, as need became evident and opportunity was found, by a number of different people, with different needs. Hence, the PROJ.4 3D support has not been the result of neither deep geodetic, nor careful code architectural considerations. This has resulted in a library that supports only a subset of commonly occurring geodetic transformations. To be more specific: It supports any datum shift that can be completed by a combination of two Helmert shifts and a non-linear planar correction derived from interpolation in a correction grid. While this is sufficient for most small scale mapping activities, it is not at all sufficient for operational geodetic use, nor for many of the rapidly emerging high accuracy geospatial applications in agriculture, construction and transportation. To improve this situation, we have introduced a new framework for implementation of geodetic transformations, which will appear in the next release of the PROJ.4 library.

Before describing the details, let us first remark that most cases of geodetic transformations can be expressed as a series of elementary operations, the output of one operation being the input of the next. E.g. when going from UTM zone 32, datum ED50, to UTM zone 32, datum ETRS89, one must, in the simplest case, go through 5 steps:

1. Back-project the UTM coordinates to geographic coordinates
2. Convert the geographic coordinates to 3D cartesian geocentric coordinates
3. Apply a Helmert transformation from ED50 to ETRS89
4. Convert back from cartesian to geographic coordinates
5. Finally project the geographic coordinates to UTM zone 32 planar coordinates.

The homology between these steps and a Unix shell style pipeline is evident. With this as its main architectural inspiration, the primary feature of our implementation is a pipeline driver, that takes as its user supplied arguments, a series of elementary operations, which it strings together in order to implement the full transformation needed. Also, we have added a number of elementary geodetic

operations, including Helmert transformations, general high order polynomial shifts and the Molodensky transformation. In anticipation of upcoming support for full time-varying transformations, we also introduce a 4D spatiotemporal data type, and a programming interface (API) for handling this.

With these improvements in place, we assert that PROJ.4 is now well on its way from being a mostly-map projection library, to becoming an almost-generic-geodetic-transformation library.

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1. INTRODUCTION

For more than 2 decades, PROJ.4 has been the globally leading map projection library for open source (and probably also closed source) geospatial software. While focusing on mathematically well-defined 2D projections from geographical to planar coordinates,

PROJ.4 has nevertheless, since its introduction in the 1980s, provided limited support for more general geodetic datum transformations, and has gradually introduced a higher degree of support for 3D coordinate data and reference systems.

The support has, however, been implemented over a long period of time, as need became evident and opportunity was found, by a number of different people, with different needs and at different times. Hence, the PROJ.4 3D support has been the result of neither deep geodetic, nor careful code architectural considerations.

This has resulted in a library that supports only a subset of commonly occurring geodetic transformations. To be more specific: It supports any datum shift that can be completed by a combination of two Helmert shifts (to and from a pivot datum) and, potentially, also a non-linear planar correction derived from interpolation in a correction grid.

While this is sufficient for most small scale mapping activities, it is not at all sufficient for operational geodetic use, nor for many of the rapidly emerging high accuracy geospatial applications in agriculture, construction engineering, transportation and utilities. To improve this situation, we have introduced a new framework for implementation of geodetic transformations, which will appear in the next release of the PROJ.4 library.

Gerald I. Evenden (1935–2016) started the PROJ.4 project in 1983, as an implementation of material from John Snyder's work in "Map projections used by the U.S. Geological Survey" (1982) and later on "Map Projections: A working manual" (1987). Evenden completed the connection between geophysical reference frames and the purely mathematical projections by supplementing Snyder's material with some basic datum transformation functionality.

Fast-forward 35 years and PROJ.4 is everywhere: It provides coordinate handling for almost every geospatial program—open or closed source.

Today, we see a drastical increase in the need for high accuracy GNSS coordinate handling, especially in the agricultural and construction engineering sectors. This need for geodetic-accuracy transformations is not satisfied by "classic PROJ.4". But with the ubiquity of PROJ.4, we can provide these transformations "everywhere", just by implementing them as part of PROJ.4.

This paper presents the "transformation pipelines" that has been introduced to PROJ.4 recently. Transformation pipelines is a highly flexible framework that allows users of PROJ.4 to perform high precision geodetic coordinate transformations. This is achieved by dividing the complete transformation into a number of building blocks, each describing an independent step of the transformation, e.g. a datum shift expressed as a Helmert transformation.

Also included in the new transformation framework is support for spatio-temporal coordinates which brings PROJ.4 usage into the realm of dynamic reference frames, a topic that is gaining more and more traction in the geodetic community.

2. COORDINATE TRANSFORMATION IN PROJ.4 TODAY

As already mentioned PROJ.4 was originally created at the USGS in the 1980's as a pure mathematical projection library heavily inspired by Snyder (1982). In the first releases the only coordinate transformations supported were between geodetic coordinates and projected coordinates and vice versa. With the release of version 4.3 in the early 1990's PROJ.4 saw the first support for transformation between US datums (Evenden & Warmerdam, 2015). The initial datum shift support was grid based and was at the time of introduction delivered in a stand-alone application called *nad2nad* bundled with PROJ.4 (Evenden, 1995a).

Later on, in the early 2000's, the grid shifting functionality was included in the core library and the ability to do 7-parameter datum shift was added shortly after. This has been the state of affairs with regards to coordinate transformation in PROJ.4 from version 4.4.2 to version 4.9.3. In the roughly 15 years between the two releases many new projections has been added to the library, as well as other features such as the ability to express coordinate reference systems by their EPSG identifiers.

Projections and transformations in PROJ.4 are expressed as "proj-strings" which holds the parameters of a given coordinate transformation, e.g. "+proj=merc +lat_ts=56.5 +ellps=GRS80". I.e. a proj-string consists of a projection specifier, **+proj**, a number of parameters that applies to the projection and, if needed, a description of a datum shift.

We will not go into the specifics of the projections and their parameters since they are outside the scope of this paper. Over a hundred different projections are supported in PROJ.4. The most common ones have been described by Evenden (1995b).

By supplying two proj-strings to PROJ.4 it is possible to perform a coordinate transformation from one coordinate reference system to another. All coordinate transformations done in this fashion are transformed in a two-step process with WGS84 as a pivot datum. That is, the input coordinates are transformed to WGS84 geodetic coordinates and then transformed from WGS84 coordinates to the specified output coordinate reference system. Datum shifts can be described in a proj-string with the parameters +towgs84, +nadgrids and +geoidgrids. An inverse transform exists for all three and is applied if specified in the input proj-string.

The most common is +towgs84, which is used to define a 3- or 7-parameter Helmert shift from the input reference frame to WGS84. Exactly which realization of WGS84 is not specified, hence a fair amount of uncertainty is introduced in this step of the transformation.

With the +nadgrids parameter a non-lineaer planar correction derived from interpolation in a correction grid can be applied. Originally this was implemented as a means to transform coordinates between the american datums NAD27 and NAD83, but corrections can be applied for any datum for which a correction grid exists. The inverse transform for the horizontal grid shift is "dumb", in the sense that the correction grid is applied verbatim without taking into account that the inverse operation is non-linear.

Similar to the horizontal grid correction, +geoidgrids can be used to perform grid corrections in the vertical component. Both grid correction methods allow inclusion of more than one grid in the same transformation.

3. GEODETIC TRANSFORMATIONS

Modern geodetic coordinate reference frames are typically based on GNSS observations, and hence almost perfect with respect to scale, geocentric origin, and alignment with the Earth rotation axis. For older systems, based on terrestrial observations (triangulation, trilateration etc.), this is not necessarily the case: Although the deviations are typically small in terms of the dimensions of the Earth, they are huge in terms of the planning needs of a modern (or even medieval) society. For example, an alignment deviation in the order of 3 seconds-of-arc (i.e. approx. 0.001 degree) between two reference frames, results in a linear misalignment in the order of 100 m on the surface of the Earth.

So in order to integrate new and older observations, maps and coordinates, it is necessary to transform them to a common frame. Typically, this is done using Helmert transformations (also known as similarity transformations).

3.1. The Helmert transformation

The 3D Helmert transformation maps 3D geocentric-cartesian coordinates from one reference frame to another through a combination of a 3D translation, a 3D rotation, and a common scaling factor, hence needing 7 parameters: 3 translations, 3 rotations, and a scaling factor.

Typically, the rotations are in the order of 1 second-of-arc, the translations in the order of 100 m, and the scale in the order of 1 plus/minus a few parts-per-million (and hence typically given as a deviation from unity).

Helmert transformations between two reference frames are derived through a least squares adjustment to a set of points with coordinates given in both frames. As the functional relationship between the frames is non-linear in the parameters, the adjustment is non-trivial (although closed form expressions exist - cf. Chang (2016) and references therein). Hence, one may be tempted to use a more general affine transformation, which can be given as an expression linear in the 9 affine parameters.

There are, however, at least two good reasons for the historical geodetic preference of the Helmert transformation over the affine: First, the Helmert transformation has a straightforward geometrical interpretation, in terms of translation, rotation and scale. Second, the scale of traditional terrestrial networks was typically derived from careful measurement of one or a few network vectors, hence inherently having one scale factor only, rather than the 3 implied in a 3D affine transformation. So physically speaking, the affine transformation represents an overparameterization of the problem at hand, and may be rejected by application of the law of parsimony ("Ockham's Razor").

3.2. Tensions in geodetic networks

While a properly constructed Helmert transformation between two reference frames is optimal in the least squares sense, it is not necessarily optimal in all common use cases.

Commonly, we would prefer to have comparable accuracy everywhere in the domain of the transformation. And while this is achievable for transformations between modern GNSS based reference frames, the same is not the case for traditional terrestrial frames. This is due to the fact that reference frames defined at times predating the age of digital computers, typically suffer from tensions, related to inaccurate original measurements, and inadequate network adjustment, due to lack of computational resources.

Hence, for older reference frames, we must correct for tensions before (or after) the application of a least squares optimal Helmert transformation.

3.3. Tension correction / residual reduction

We are aware of only two tension correction methods in general use. The most widespread is the grid based NADCON method (Dewhurst, 1990), where the entire correction is implemented through interpolation in a grid giving the linear local shift from one datum to the other, totally bypassing the Helmert step.

Evidently, the NADCON method might as well be applied as a correction step in connection with a Helmert transformation, essentially splitting the transformation into a deterministic (Helmert) part and a stochastic (residual, grid interpolation) part.

A less known method is to model the network tension as a high order 2D polynomial. As the high order terms of such polynomials will typically have very small coefficients, it is important to evaluate the polynomials using a numerically stable algorithm. In the PROJ.4 implementation, we use the 2D Horner's Scheme, where the tiny high order terms are evaluated first, and summed, in order to avoid numerical underflow due to the much bigger low order terms.

3.4. Kinematic Helmert transformations

Plate tectonic motion can be modelled as a 3D geocentric rotation of plate centers. Hence transformations between global, geocentric reference frames and regional, plate fixed reference frames can be expressed as Helmert transformations. To resolve the kinematic nature of the problem, the plate motion is expanded to linear order, and modelled using a time varying Helmert transformation, implemented as an ordinary 7 parameter version, augmented with a velocity term for each parameter, resulting in 14 parameters in total.

The 14 parameter Helmert transformation is the workhorse for transformation between modern GNSS based reference frames, and an obvious prerequisite for a contemporary transformation package.

3.5. Other reference frame transformation methods

At least two other methods are in general use for reference frame transformation: The Molodensky transformation, and the generic complex polynomial. And while they may be considered approximation methods, they are, by virtue of being used in some officially endorsed (i.e. "exact by definition") transformations, mandatory elements of any collection of transformation methods worth its salt.

3.5.1. The Molodensky transformation

The Molodensky transformation resembles a Helmert transformation with zero rotations and a scale of unity, but converts directly from geodetic coordinates to geodetic coordinates, without the intermediate shifts to and from cartesian geocentric coordinates, associated with the Helmert transformation.
The Molodensky transformation is simple to implement and to parameterize, requiring only the 3 shifts between the input and output frame, and the corresponding differences between the semi-major axes and flattening parameters of the reference ellipsoids.

Due to its algorithmic simplicity, it was popular prior to the ubiquity of digital computers. Today, it is mostly interesting for historical reasons, but nevertheless indispensable due to the large amount of data that has already been transformed that way.

3.5.2. Complex polynomials

Mathematically, conformal mappings can be implemented, and hence approximated, as complex polynomial series. Hence, as demonstrated by e.g. Lippus and Oja (2012), complex polynomials are a highly useful tool for generic (but conformal) geodetic transformations, including transformations between reference frames, reprojections between different conformal projections (e.g. between Lambert Conformal Conic and Transverse Mercator projections), and combinations of both cases.

4. TRANSFORMATION PIPELINES

As already mentioned, the Molodensky transformation converts directly from geodetic coordinates in one datum, to geodetic coordinates in another datum, while the (typically more accurate) Helmert transformation converts from 3D cartesian to 3D cartesian coordinates.

So when using the Helmert transformation one typically needs to do an initial conversion from geodetic to cartesian coordinates, and a final conversion the other way round, to arrive at the desired result. Fortunately, this three-step compound transformation has the attractive characteristic that each step depends only on the output of the immediately preceding step. Hence, we can build a geodetic-to-geodetic Helmert transformation by tying together the outputs and inputs of 3 steps (geodetic-to-cartesian → Helmert → cartesian-to-geodetic), pipeline style.

In a recent extension to the PROJ.4 transformation library, we have implemented a pipeline driver, making this kind of chained transformations possible. The implementation is compact, consisting of just one pseudo-projection (called *pipeline*), which takes as its arguments strings of elementary projections (note: "projection" is the, slightly misleading, PROJ.4 term used for any kind of transformation).

The pipeline pseudo projection is supplemented by a number of elementary transformations, implementing a number of the geodetic algorithms mentioned in the previous section, all in all providing a framework for building high accuracy solutions for a wide spectrum of geodetic tasks. As a first example, let us take a look at the iconic "geodetic → Cartesian → Helmert → geodetic" case. In PROJ.4 it can be implemented as

```
proj=pipeline
step  proj=cart     ellps=intl

step  proj=helmert
    x=-81.0703 y=-89.3603 z=-115.7526
   rx=-0.48488 ry=-0.02436  rz=-0.41321   s=-0.540645

step  proj=cart inv ellps=GRS80
```

Example 1. Transformation pipeline from geodetic (ED50) to geodetic (ETRS89)

The pipeline can be expanded at both ends to accommodate whatever coordinate type is needed for input and output: In example 2, we transform from the deprecated Danish System 45, a 2D system with some tension in the original defining network, to UTM zone 33, ETRS89. The tension is reduced using a polynomial transformation (the init=./s45b... step, s45b.pol is a file containing the polynomial coefficients), taking the S45 coordinates to a technical coordinate system (TC32), defined to represent "UTM zone 32 coordinates, as they would look if the Helmert transformation between ED50 and ETRS89 was perfect". The TC32 coordinates are then converted back to geodetic(ED50) coordinates, using an inverse UTM projection, further to cartesian(ED50), then to cartesian(ETRS89), using the relevant Helmert transformation, and back to geodetic(ETRS89), before finally being projected onto the UTM zone 33, ETRS89 system.

All in all a 6 step pipeline, implementing a transformation with centimeter level accuracy from a deprecated system with decimeter level tensions.

```
<s45b>
proj=pipeline

step  init=./s45b.pol:s45b_tc32
```

```
step  proj=utm inv   ellps=intl zone=32
step  proj=cart      ellps=intl

step  proj=helmert
    x=-81.0703 y=-89.3603 z=-115.7526
   rx=-0.48488 ry=-0.02436  rz=-0.41321   s=-0.540645

step  proj=cart inv ellps=GRS80
step  proj=utm ellps=GRS80  zone=33
```

Example 2. Transformation pipeline from Danish System45 to ETRS89/UTM33

With the pipeline framework spatiotemporal transformation is possible. This is possible by leveraging the recently added time dimension in PROJ.4 that enables 4D coordinates (three spatial components and one temporal component) to be passed through a transformation pipeline. In example 3 a transformation from ITRF93 to ITRF2000 is defined. The temporal component is given as GPS weeks in the input data, but the 14-parameter Helmert transform expects temporal units in decimalyears. Hence the first step in the pipeline is the *unitconvert* pseudo-projection that makes sure the correct units are passed along to the Helmert transform. Most parameters of the Helmert transform are taken from Altamimi & Boucher (2002), except the *epoch* which is the epoch of the transformation. The default setting is to use "coordinate frame" convention of the Helmert transform, but "position vector" convention can also be used. The last step in the pipeline is converting the coordinate timestamps back to GPS weeks.

```
proj=pipeline

step  proj=unitconvert t_in=gps_week t_out=decimalyear

step  proj=helmert
    x=0.0127       y=0.0065      z=-0.0209  s=0.00195
    rx=0.00039     ry=-0.00080   rz=0.00114
    dx=-0.0029     dy=-0.0002    dz=-0.0006  ds=0.00001
    drx=0.00011    dry=0.00019  drz=-0.00007
    epoch=1988.0

step  proj=unitconvert t_in=decimalyear t_out=gps_week
```

Example 3. Transformation pipeline that translates between ITRF93 and ITRF2000 coordinates for variable observation epochs, in GPS weeks, of input data (Altamimi & Boucher, 2002).

5. DISCUSSION

The introduction of transformation pipelines in PROJ.4 opens up a new world of possibilities in geospatial software. Amongst them is implementation of dynamic reference frames (DRF). The spatiotemporal capabilities added is a first step towards implementing dynamic reference frames, such as the recently announced ATRF in Australia (ICSM, 2016), in mainstream GIS applications.

The technical details of the new Australian datum have not yet been released but it is certain that time-varying coordinates will be necessary in some form or another. Within a dynamic reference frame well-defined coordinate transformations are even more important than previously. In a dynamic reference frame it is not only necessary to transform coordinate between reference frames, it is also necessary to transform coordinates to the same epoch within the dynamic reference frame in order to keep the coordinates consistent.

Since coordinates vary in time, it is necessary to transform DRF-coordinates to a common epoch before they can truly be compared. This might not be necessary in small-scale mapping but in situations where high-precision coordinates are crucial, the need for transformation within the reference frame is evident. The concept is not unlike that of RTK GNSS where coordinates are propagated through time via a kinematic model. The difference between the two scenarios is the order of operation. In the case of a dynamic reference frame, the transformation happen client-side whereas in the case of RTK GNSS the transformation is performed by the network operator and the transmitted coordinates refer to a conventional static reference frame.

Another possibility with the new transformation framework in PROJ.4 is realizing the reference frame and coordinate model of the updated version of the ISO-19000 standardization series. The ISO 19162:2015 (ISO/TC 211, 2015) standard is vital in geospatial software because it standardizes how geospatial information is shared. One essential part of the standard is how coordinate reference systems are described in the metadata of files containing geospatial data. The standard defines what is referred to as Well-Known Text, or WKT in short. WKT exists in two versions, WKT and WKT2. So far WKT2 has not been implemented in practice and all geospatial data files describe the coordinate reference systems with WKT. The reason for that is that in WKT it is possible to describe the reference frame of a dataset as a set of 7 Helmert transformation parameters that transforms the data to WGS84. This possibility has been removed in WKT2. Since PROJ.4 has not been able to do datum shifts in other ways than by utilizing WGS84 as a pivot datum, it has not been possible to implement WKT2 in practice in the open source software stack. With the transformation pipelines in PROJ.4 we now have the geodetic foundation in place that allows broader acceptance of WKT2 in the geospatial community.

6. FUTURE WORK

Even though we have come a long way in terms of improving coordinate transformations in PROJ.4, there is still work to do before the changes will benefit the average GIS user. PROJ.4 is today already in widespread use in geospatial software, mainly in the open source system but it is also used in some of the more notable geospatial software packages.

It is important to note that the new capabilities in PROJ.4 are best utilized via the new API that has been introduced with the transformation pipelines. The new API enables use of the time dimension, as well as other auxiliary observation data that might be needed for a specific transformation. It is also tailor-made for the transformation pipeline framework, in contrast to the old API where transformation pipelines will only work to some extent, as they have to be shoehorned in by exploiting the *latlong* pseudo-projection.

So before high-precision spatiotemporal coordinate transformation can become available in mainstream GIS applications the downstream projects will have to adopt the new API. Adopting a new API is obviously not something that will happen overnight and it is a process that should not be taken lightly by software developers. The old API has stood the test of time and has proven to be very stable. We hope that the new API can deliver the same stable performance and that the open source geospatial community can see the benefit that the new API brings to the users.

If the new API is adopted in these software packages, a wide range of national coordinates systems, and more importantly transformations to and from them, that has previously been impossible to use in most GIS applications will suddenly become available to more users. An example of this is the Danish System 34/45 (described in section 4) which previously has required software vendors to include the transformation library TrLib released by Danish authorities. Only a few software vendors has bundled TrLib with their software resulting in users having to resort to transformation applications external to their main GIS working environment. This is not exclusive to Denmark and similar situations are seen across the world.

In a world where we increasingly rely on satellite positioning systems the need for good coordinate transformations is of great significance. Most users of such systems are unaware of the geodetic implications that in the end result in precise positioning. A precise coordinate is taken for granted without considering how the coordinate came to be. This is to be expected of casual users, but unfortunately the same can also be said about many professional users of geospatial data. Users that very well could make fatal mistakes based on too high expectations for the precision of the data that they are using. The new pipeline framework in PROJ.4 is very verbose in the way that transformations are defined, and can serve as a good educational tool as well as a good transformation library.

7. CONCLUSION

> "*A* ***lot of luck*** *to whoever want to put together the computational part of the datum shift software." – Gerald I. Evenden, 2000.*

These words were uttered in a mailing list discussion about improving datum shifting in PROJ.4. While some support for datum shifts were introduced by Frank Warmerdam shortly after, it took more than 15 years before someone took on the task of adding more complete geodetic datum shifting capabilities. Nevertheless we believe we have now largely succeeded in the task: The projection library has been turned into a full-fledged generic geodetic transformation library.

This allows for a range of new possibilities in mainstream GIS applications, e.g. better support for old systems defined by transformations that were previously not available in PROJ.4. Also, by introducing a generic framework of transformation building blocks, we have provided the essentials for support and dissemination of future fully dynamic spatiotemporal reference systems.

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BIOGRAPHICAL NOTES

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