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**THE GRAVIMETRIC QUASIGEOID MODEL OVER UGANDA**



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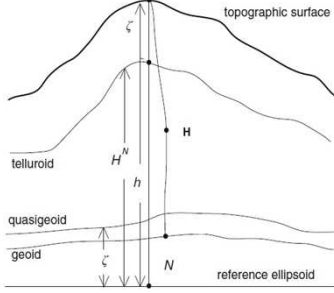
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**INTRODUCTION**

- Need for geoid/quasigeoid model?---GNSS-HEIGHT DETERMINATION**



$$H = h - N \quad H^N = h - \zeta$$



$H$  = orthometric height     $N$  = geoid height     $\zeta$  = height anomaly/quasigeoid height  
 $h$  = ellipsoidal height     $H^N$  = Normal height



The diagram illustrates the relationship between different height systems. It shows a topographic surface, a telluroid, a quasigeoid, a geoid, and a reference ellipsoid. The topographic surface is the highest, followed by the telluroid, quasigeoid, geoid, and reference ellipsoid. The height from the topographic surface to the telluroid is labeled  $\zeta$ . The height from the topographic surface to the geoid is labeled  $H$ . The height from the topographic surface to the reference ellipsoid is labeled  $h$ . The height from the geoid to the reference ellipsoid is labeled  $N$ . The height from the quasigeoid to the reference ellipsoid is labeled  $H^N$ . The height from the quasigeoid to the geoid is labeled  $\zeta'$ .

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### INTRODUCTION

#### Quasigeoid determination

- Directly using Stokes formula/modification

$$\zeta_p = \frac{R}{4\pi\gamma_\sigma} \iint S(r_p, \psi) \Delta g^* d\sigma$$

- Indirectly

$$\zeta_p = N_p + (\zeta - N)$$

$(\zeta - N)$  = quasigeoid-geoid separation



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### INTRODUCTION

#### Quasigeoid determination

- Based on the Uganda Gravimetric Geoid Model 2014—KTH method

$$\bar{N}^{L,M} = \frac{R}{4\pi\gamma_{\sigma_0}} \iint S^L(\psi) \Delta g d\sigma + c \sum_{n=0}^M (Q_n^L + s_n) \Delta g_n^{GGM} + \delta N_{comb}^T + \delta N_{dwc} + \delta N_{tot}^a + \delta N_{tot}^c$$

- $\sigma_0$  = spherical cap
- $R$  = mean Earth radius
- $\gamma$  = mean normal gravity on reference ellipsoid
- $S^L(\psi)$  = modified Stokes' function
- $M$  = maximum degree of GGM
- $L$  = maximum degree of modification
- $Q_n^L$  = Molodensky truncation coefficients



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### Data used for the determination of the geoid model

- 7839 terrestrial gravity data from BGI
- World Gravity Map 2012 surface gravity anomalies –BGI
- SRTM3 DEM—CGIAR-CSI
- GOCE-only GGM—GO\_CONS\_GCF\_2\_TIM\_R5 (maximum degree=280)
- 10 GNSS/levelling points

### Internal & external accuracy assessment of UGG2014

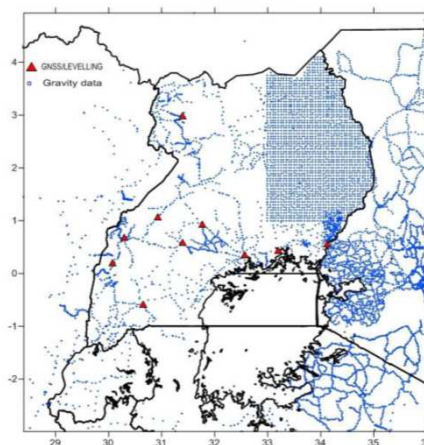
- Internal accuracy= 11.5 cm
- External accuracy before & after 4-parameter fitting = 11.6 cm and 7.4 cm



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### Data used for the determination of the geoid model





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**Determination of the quasigeoid-geoid separation**

- Approximate formula (Heiskanen & Moritz, 1967)

$$\zeta - N \approx -\frac{\Delta g_B}{\gamma_0} H$$

$\Delta g_B$  = Bouguer gravity anomaly

$H$  = topographic height (SRTM3)

$\gamma_0$  = normal gravity at latitude 45°

- Strict formula (Sjöberg, 2006;2010)

$$\zeta - N = \frac{T(r_p, \Omega)}{\gamma_Q(\varphi)} - \frac{T^*(r_g, \Omega)}{\gamma_0(\varphi)} + \frac{V'_{bias}(r_p, \Omega)}{\bar{\gamma}(\Omega)}$$



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**Determination of the quasigeoid-geoid separation**

- Comparison of the results of approximate and strict formula for QGGS (Units: m)

Formula	Min.	Max.	Mean	Std.	RMS
Approximate	-0.08	<b>0.72</b>	0.16	0.08	0.17
Strict	-0.05	<b>3.35</b>	0.17	0.19	0.25

- Topographic bias in the strict formula ---harmonic series expansion

$$V'_{bias}(r_p, \Omega) = 2\pi G \rho_0 \sum_{n=0}^{n_{max}} \sum_{m=-n}^n \left( H_{nm}^2 + \frac{2}{3R} H_{nm}^3 \right) Y_{nm}(\Omega)$$



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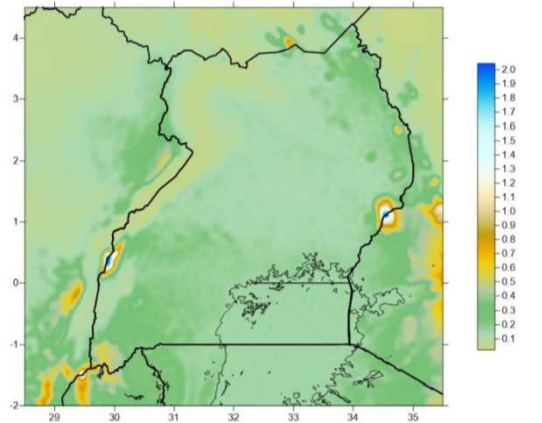




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**Topographic bias over Uganda**



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**UGQ2014 & its evaluation**

$$\zeta_p = N_p + (\zeta - N)_i$$

GNSS/levelling residuals over 10 points before and after the 4-parameter fit (units: cm)

Formula		Min	Max	Mean	Std.	RMS
<b>Approximate</b>	Before	2.56	51.41	24.10	12.74	<b>29.96</b>
	After	-20.46	13.80	0.0	10.90	<b>10.34</b>
<b>Strict</b>	Before	-30.54	14.56	-9.29	13.18	<b>15.57</b>
	After	-9.96	12.91	0.0	6.65	<b>6.31</b>

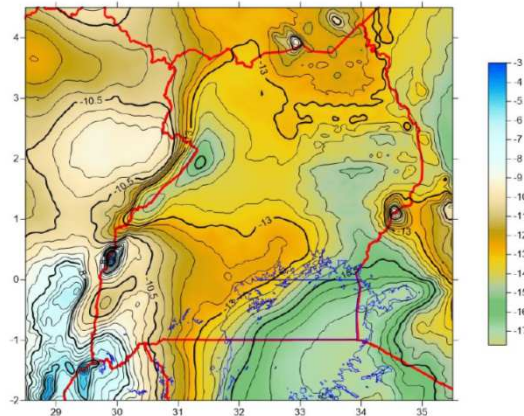




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### UGQ2014



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### Conclusion

- Standard errors of ellipsoidal heights = 2.2 cm
- Standard errors of normal-orthometric heights = 1.0 cm
- **Standard error of UGQ2014 before & after fitting are 15.4 cm and 5.8 cm**
- **Satisfactory given the poor quality & quantity of terrestrial gravity data**
- **Comparison of approximate & strict formulas shows that --- introduce errors of 2.6 m in the QGGS --- 35 cm in the final quasigeoid heights**
- **Future work ---**
  - GNSS/levelling observations to create a more homogeneous dataset
  - Government– airborne gravimetry for better quality gravity data



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