

# Finite Elements Method and its Space Application in the Study of Movements of GNSS BULiPOS Reference Stations in Bulgaria

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**Keywords:** Finite Elements, GPS, Movement, Deformations

## SUMMARY

The main objective of the work is the study of motion of GNSS BULiPOS reference stations in Bulgaria by applying Finite Elements Method (FEM) for the space. A development of the theoretical basis of the Finite Element Method for the spatial case is proposed. The developed model is applied to study the spatial changes of GNSS reference stations from BULiPOS network in Bulgaria. GPS data from one-week periods in five years 2009-2013 are used and processed with Berne software, Version 5.0. Obtained estimations of Cartesian coordinates of the stations are used in the proposed FE model. Vectors of displacement for each apex of each formed finite element (triangle) change the lengths of the sides of each triangle and relative deformations of the sides of each triangle are calculated. The results obtained have been analyzed and areas of compression and extension have been inferred. An assessment of the effectiveness of the proposed method is made.

## РЕЗИОМЕ

Основната цел на предлаганата разработка е изследване на изменение на положението на GNSS BULiPOS референтните станции чрез прилагане на метода на крайните елементи (МКЕ) за пространството. Предлага се развитие на теоретичната основа на метода на крайните елементи за пространствения случай. Същата е приложена за изследване на пространствените изменения на GNSS станциите от BULiPOS референтната мрежа в България. Използвани са GPS данните от едноседмични периоди от пет години - 2009-2013, които са обработени с бернския софтуер, версия 5.0. Получените оценки на пространствените координати на станциите са използвани в предложението ФЕ модел. За всеки краен елемент (триъгълник) са изчислени векторите на преместване на всеки един от върховете, изменението на дължините на страните на триъгълника и относителните деформации на страните на триъгълника. Получените резултати са анализирани и са определени зоните на свиване и разтягане, като се прави оценка на ефективността на предлагания метод.

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## 1. INTRODUCTION

The territory of Bulgaria characterizes with active tectonics and seismotectonics. A number of geological, geophysical and geodetic investigations demonstrate the recent activity of the region and tried to give a reasonable and adequate interpretation of the obtained results [4], [7], [8], [10], [11], [13], [14], [17].

The advanced Global Navigation Satellite Systems (GNSS) have been recently used for geodetic observations and determination of the earth crust movements in millimetre level. Velocity vectors of located GNSS stations can be estimated in global and in local geodetic coordinate systems and their behaviour can be studied. For precise determination of station movements they mostly operate as GNSS permanent stations and long-term data are processed with sophisticated software. Except station behaviour also surface deformations are of interest for the earth movements of blocks or large areas. This paper is an attempt to contribute to the geokinematics of the territory of Bulgaria by means of Finite Elements Model (FEM) using estimated GNSS station coordinates in different time periods.

## 2. FINITE ELEMENTS METHOD FOR THE SPACE

Finite Elements Models recently are successfully used in the analysis of movements of stations from GNSS data processing in order to be obtained strain tensors and strain accumulation [1], [2], [6], [9].

In this work the FEM is developed and applied for deformation analysis in the space, for determination of linear deformations of baselines between GNSS stations. The estimated three-dimensional coordinates of GNSS stations are used as input data by this approach. The finite elements (triangles) for different observational epochs are formed in such a way that they are not overlapped. Similar approach is applied in [6].

As an appropriate method for deformation analysis based on GNSS data for large territories here it is proposed a FE model for the space where the baseline vectors between stations in space are projected onto the ellipsoid used. The estimated space positions of investigated stations obtained from GNSS data processing are first projected onto the WGS84 ellipsoid and then the finite elements are configured by using ellipsoidal chords (baselines on the ellipsoid) between projected station positions. The FEM theory developed and applied for deformation analysis is presented in the next section.

### 2.1 Transformation of Cartesian three-dimensional coordinates into ellipsoidal coordinates

Ellipsoidal geodetic latitude  $\varphi$  and ellipsoidal geodetic longitude  $\lambda$  of GNSS stations are calculated from the estimated Cartesian station coordinates  $X$ ,  $Y$ ,  $Z$  by the following formulas

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$$\lambda = \arctg(Y / X) \quad (1)$$

$$\operatorname{tg} \varphi = k \cdot \operatorname{tg} \varphi' \quad (2)$$

where

$$\operatorname{tg} \varphi' = \frac{Z(1+e^2)}{\sqrt{X^2+Y^2}}, \quad k = \left(1 + \frac{H}{N} e^2\right)^{-1}. \quad (3)$$

The ellipsoidal height  $h$  of stations is calculated as

$$h = \frac{A}{N'} (A - a), \quad (4)$$

where

$$A = \sqrt{X^2 + Y^2 + Z^2(1+e^2)}, \quad N' = \sqrt{X^2 + Y^2 + Z^2(1+e'^2)^2} \quad (5)$$

The obtained accuracy of ellipsoidal height and latitude are respectively -  
 $m_h \leq \pm 0.1 \text{ mm}$ ,  $m_\varphi \leq \pm 0.000\,001''$  if the ellipsoidal height is  $h < 10\,000 \text{ m}$ .

## 2.2 Transformation of ellipsoidal coordinates into Cartesian three-dimensional coordinates of projected onto the ellipsoid stations

The transformation of ellipsoidal coordinates  $\varphi$ ,  $\lambda$  and  $h$  into Cartesian coordinates of projected onto the ellipsoid stations is accomplished by the known formulas:

$$\begin{aligned} X_o &= N_o \cdot \cos \varphi \cdot \cos \lambda, \\ Y_o &= N_o \cdot \cos \varphi \cdot \sin \lambda, \\ Z_o &= N_o \cdot (1 - e^2) \cdot \sin \varphi. \end{aligned} \quad (6)$$

where  $e$  is the first eccentricity of the ellipsoid,  $N_o$  is the transversal radius of curvature, calculated by the formula:

$$N_o = \frac{a}{\sqrt{(1 - e^2 \sin^2 \varphi)}} \quad (7)$$

Let  $X_o$ ,  $Y_o$ , and  $Z_o$  are the Cartesian coordinates of station  $P_o$ , which is the pierced of the normal line with the ellipsoid. Point  $P_o$  must satisfy both the equation of meridian ellipse and the equation of the normal line though the point  $P$  to this ellipse, namely

$$r_o^2 + Z_o^2(1+e'^2) = a^2 \quad (8)$$

$$\frac{Z - Z_o}{r - r_o} = \frac{Z_o(1+e'^2)}{r_o}, \quad (9)$$

where  $r_0$  is the radius of the parallel, which passes through the point  $P_0$  and  $r$  is the distance from point  $P$  to the  $Z$  axis and they are calculated as follows

$$r_0 = \sqrt{X_0^2 + Y_0^2}, r = \sqrt{X^2 + Y^2}. \quad (10)$$

After elimination of  $Z_0$  from equations (8) and (9) an equation of 4<sup>th</sup> degree is obtained:

$$r_o^4 + A.r_o^3 + B.r_o^2 + C.r_o + D = 0, \quad (11)$$

where

$$A = -\frac{2r}{e^2}, B = \frac{r^2}{e^4} + \frac{z^2}{e^2 e'^2} - a^2, C = -Aa^2, D = \frac{AC}{4}. \quad (12)$$

In general, the possibilities are as follows. The equation (11) has four real roots if the point  $P$  lies inside the asteroid. If the point  $P$  lies on the asteroid itself, the real roots are 3, one is double. Two roots are obtained if the point is outside the asteroid. Two real roots are obtained as well if the point  $P$  coincides with the horn point of the asteroid.

In case of the earth ellipsoid, which has a very small eccentricity, points are always outside the asteroid. They are even outside the ellipsoid, which means that the equation always has two real roots, from which only the one root is the solution of the equation.

The Ferrari method [12] is the most convenient to be solved the equation (11) as it is a method for reducing the solution of an equation of degree 4 to the solution of one cubic and two quadratic equations. First the following cubic equation is solved

$$\eta^3 - B\eta^2 + (AC - 4D)\eta - A^2D + 4BD - C^2 = 0, \quad (13)$$

or

$$\eta^3 + p.\eta^2 + q.\eta + s = 0 \quad (14)$$

where

$$p = -B, q = AC - 4D, s = -(A^2 - 4BD + C^2) \quad (15)$$

In all cases, a real root must be found. The root  $\eta$  is calculated after obtaining the first real root  $\xi$ .

$$\eta = \xi - \frac{q}{3}. \quad (16)$$

The four roots of the equation (13) are found as roots of two quadratic equations:

$$r_o^2 + \left( \frac{A}{2} r_o \pm \sqrt{\frac{A^2}{4} - B + \eta_1} \right) . r_o^2 + \left( \frac{\eta_1}{2} \pm \sqrt{\frac{\eta_1^2}{4} - D} \right) = 0. \quad (17)$$

Apparently the real roots will be two, of which only one will lead to point  $P_0$ , which is nearest.

Coordinates of point  $P_0$  are calculated by using the resulting root  $r_0$

$$X_o = X \frac{r_o}{r}, Y_o = Y \frac{r_o}{r}, Z_o = Z \frac{r_o}{r}, \quad (18)$$

where  $r = \sqrt{X^2 + Y^2 + Z^2}$ .

The resulting coordinates  $X_o, Y_o, Z_o$  are used to be obtained geodetic geographic coordinates and the ellipsoidal height of point  $P$  by the following relationships:

$$B = \arctg \left[ \frac{Z_o(1+e'^2)}{r_o} \right],$$

$$L = \arctg \left( \frac{Y_o}{X_o} \right), \quad (19)$$

$$H = \sqrt{(X - X_o)^2 + (Y - Y_o)^2 + (Z - Z_o)^2}.$$

### 2.3 Determination of azimuths of the sides of ellipsoidal chord triangles

The azimuths  $\alpha_{i,k}$  of the ellipsoidal triangle chords between triangle apexes  $i$  and  $k$  are obtained from the following relationship:

$$\operatorname{tg} \alpha_{i,k} = \frac{(\vec{b} \cdot \vec{r})}{(\vec{t} \cdot \vec{r})} \quad (20)$$

where

$b(-\sin\lambda, \cos\lambda, 0)$  and  $t(-\sin\varphi \cdot \cos\lambda, -\sin\varphi \cdot \sin\lambda, \cos\varphi)$  are vectors  $\vec{b}$  and  $\vec{t}$  of the natural trihedron on the ellipsoid and,  $r_{i,k}(X_k - X_i, Y_k - Y_i, Z_k - Z_i)$  is the radius-vector  $r_{i,k}$ .

### 2.4 Determination of lengths of the ellipsoidal triangle chords

The lengths of the ellipsoidal triangle chords between triangle apexes  $i$  and  $k$  are determined by the known relationships for the space, e.g.

$$S_{i,k} = \sqrt{(X_k^o - X_i^o)^2 + (Y_k^o - Y_i^o)^2 + (Z_k^o - Z_i^o)^2}. \quad (21)$$

### 2.5 Determination of vectors of displacements at the apexes of the triangles (finite elements) and deformations of triangle sides

Coordinate components of the vectors of displacement are obtained as follows:

$$\begin{aligned} dx &= X - X' \\ dy &= Y - Y', \end{aligned} \quad (22)$$

$$dz = Z - Z'$$

where  $(X, Y, Z)$  and  $(X', Y', Z')$  are the coordinates of triangle apexes  $(i, j, k)$  in the observational epochs  $t$  and  $t'$ .

Linear deformations of triangle sides are obtained from the following relationships

$$m_{i,j} = \frac{S'_{i,j}}{S_{i,j}} - 1, \quad m_{i,k} = \frac{S'_{i,k}}{S_{i,k}} - 1, \quad m_{j,k} = \frac{S'_{j,k}}{S_{j,k}} - 1, \quad (23)$$

where  $m_{i,j}$ ,  $m_{i,k}$  and  $m_{j,k}$  are the relative linear triangle side deformations,  $S$  and  $S'$  are the lengths of triangle sides in the observational epochs  $t$  and  $t'$ .

The proposed approach is appropriate for study of geodynamical phenomena of large territories where the finite elements are with long baselines.

### 3. GPS DATA PROCESSING OF BULIPOS REFERENCE NETWORK

The BULiPOS GNSS reference network [15], [16], [18] covering evenly the territory of Bulgaria (Figure 1) has been used for demonstration the proposed Finite Elements Model in space for large territories. Locations of BULiPOS permanent GNSS stations are shown in figure 1 on the tectonic map after Dabovski, Zagorchev, 2009, [5].

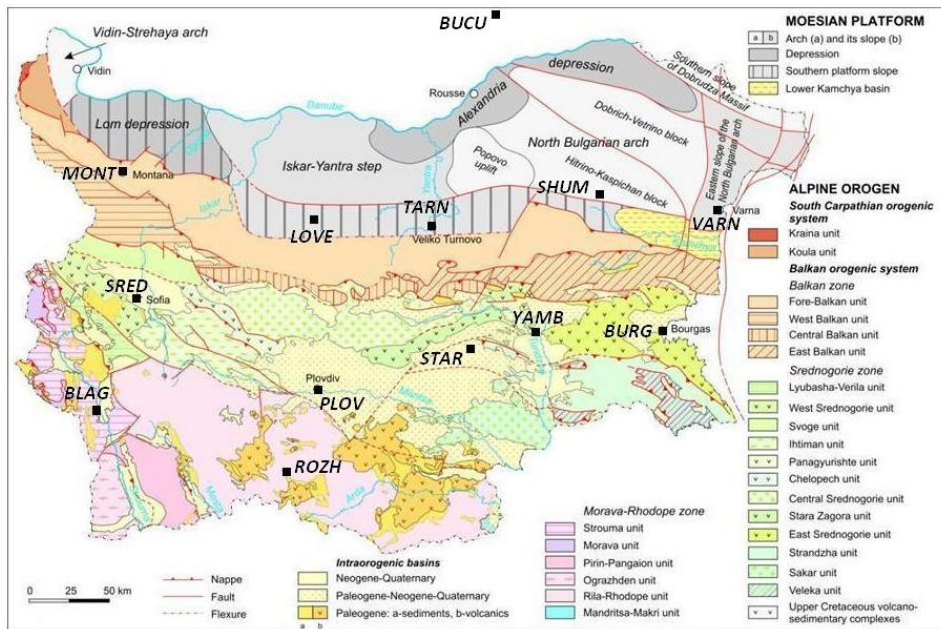


Fig. 1. BULiPOS GNSS reference network coverage (tectonic map after Dabovski, Zagorchev, 2009)

One week GPS data from each of the five years 2009 - 2013 of BULiPOS network stations have been processed with Bernese software, version 5.0. Ten IGS stations have been

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involved in the processing. By reason of consistency the same IGS stations have been used for datum definition in each of the five years and for each year their quality has been tested by Helmert transformation. Data from 2009, 2010 and from 2011 have been processed in ITRF2005, and data from 2012 and 2013 have been processed in ITRF2008. The estimated Cartesian coordinates of the stations in two time frames have been transformed in ETRF2000 by applying ETRF components of the Eurasia plate rotation pole [3]. These relative to the Eurasia stable plate station coordinates have been used in the finite elements model.

#### 4. FEM APPLICATION FOR THE BULIPOS REFERENCE NETWORK

The finite elements have been configured using all BULiPOS stations and the nearest three IGS stations – BUCU, ISTA, ORID from all together 10 IGS stations, as it is shown in figure 2.

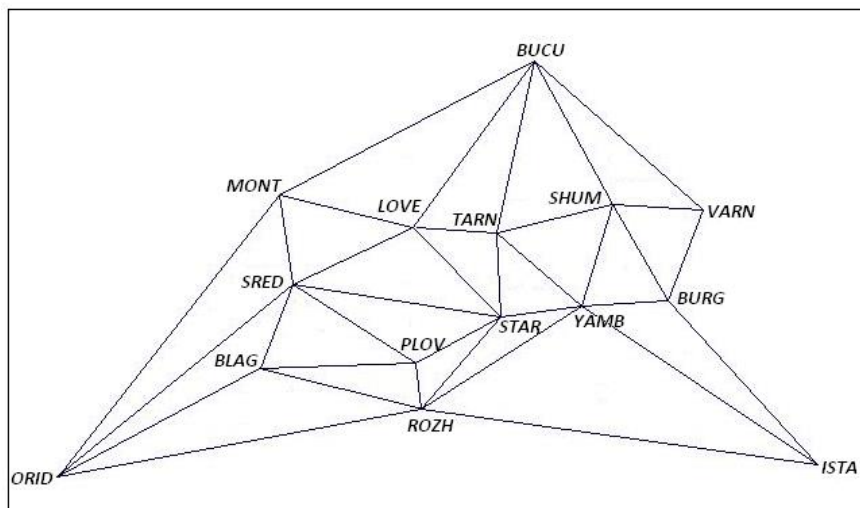


Fig. 2. Configured finite elements

All together 21 finite elements (triangles) have been formed. The ETRF2000 coordinates  $X$ ,  $Y$ ,  $Z$  of all stations in each year have been transformed into ellipsoidal coordinates  $\varphi$ ,  $\lambda$ ,  $h$  according to formulas (1), (2), (4) in section 2.1. Then the obtained ellipsoidal coordinates  $\varphi$ ,  $\lambda$ ,  $h$  of the stations have been transformed into Cartesian three-dimensional coordinates  $X_0$ ,  $Y_0$ ,  $Z_0$  onto the ellipsoid using formulas (6) in section 2.2. The latter have been used for determination the ellipsoidal chords (baselines between stations on the ellipsoid), which are actually triangle sides of every finite element in each year. According to the formula (23) linear deformations of all triangle sides have been determined using obtained lengths of triangle sides from all five years in all combinations between five year's results. Results from the 3 most representative combinations (cases) are given in Tables 1 and in Table 2. They present the linear extensions or compressions of triangle sides for time span of four, three and two years.

Table 1. Linear deformations of triangle sides of configured finite elements

Time span	Relative deformations of the lengths of triangle sides, [*E-7]											
	Finite element – finite element apex – side – linear deformation											
	BUCU MONT LOVE			BUCU LOVE TARN			BUCU TARN SHUM			BUCU SHUM VARN		
	A	B	C	A	B	C	A	B	C	A	B	C
AB	AC	BC	AB	AC	BC	AB	AC	BC	AB	AC	BC	
<b>2009-2013</b>	-0,1073	-0,1496	-0,0329	-0,1496	-0,0138	0,3330	-0,0138	0,1537	-0,4377	0,1537	-0,4524	-0,7596
<b>2010-2013</b>	-0,1516	-0,2135	-0,1258	-0,2135	-0,0664	0,3879	-0,0664	0,0466	-0,5462	0,0466	-0,2973	0,0383
<b>2011-2013</b>	-0,1440	-0,2413	-0,2944	-0,2413	-0,0660	0,5538	-0,0660	-0,0680	-0,6455	-0,0680	-0,3125	0,0924
	MONT SRED LOVE			SRED STAR PLOV			SRED LOVE STAR			SRED PLOV BLAG		
	A	B	C	A	B	C	A	B	C	A	B	C
	AB	AC	BC	AB	AC	BC	AB	AC	BC	AB	AC	BC
	AB	AC	BC	AB	AC	BC	AB	AC	BC	AB	AC	BC
<b>2009-2013</b>	0,4530	-0,0329	-0,4693	0,3117	-0,5109	0,7114	-0,4693	0,3117	1,0306	-0,5109	0,6666	0,5375
<b>2010-2013</b>	0,4180	-0,1258	-0,4079	0,2018	-0,5860	0,6471	-0,4079	0,2018	0,7863	-0,5860	0,4741	0,4239
<b>2011-2013</b>	0,0873	-0,2944	-0,3985	0,2251	-0,2993	0,7312	-0,3985	0,2251	0,7638	-0,2993	0,5811	0,4139
	SRED BLAG ORID			ORID BLAG ROZH			BLAG PLOV ROZH			LOVE TARN STAR		
	A	B	C	A	B	C	A	B	C	A	B	C
	AB	AC	BC	AB	AC	BC	AB	AC	BC	AB	AC	BC
	AB	AC	BC	AB	AC	BC	AB	AC	BC	AB	AC	BC
<b>2009-2013</b>	0,6666	0,1385	-0,2749	-0,2749	0,0060	0,1313	0,5375	0,1313	-0,1676	0,3330	1,0306	0,0151
<b>2010-2013</b>	0,4741	0,1220	-0,2341	-0,2341	0,0009	0,1332	0,4239	0,1332	0,2098	0,3879	0,7863	-0,3106
<b>2011-2013</b>	0,5811	0,0866	-0,2737	-0,2737	-0,0609	0,1710	0,4139	0,1710	0,2252	0,5538	0,7638	-0,6295

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Table 2. Linear deformations of triangle sides of configured finite elements

Time span	Relative deformations of the lengths of triangle sides, [ $\cdot 10^{-7}$ ]											
	Triangle – apex – side – deformation											
	ROZH PLOV STAR			ROZH STAR YAMB			TARN STAR YAMB			SHUM YAMB BURG		
	A	B	C	A	B	C	A	B	C	A	B	C
<i>AB</i>	<i>AC</i>	<i>BC</i>	<i>AB</i>	<i>AC</i>	<i>BC</i>	<i>AB</i>	<i>AC</i>	<i>BC</i>	<i>AB</i>	<i>AC</i>	<i>BC</i>	
<b>2009-2013</b>	-0,1676	0,3515	0,7114	0,3515	0,0055	-1,3159	0,0151	-0,0534	-1,3159	-0,6112	-0,0963	-0,1812
<b>2010-2013</b>	0,2098	0,3871	0,6471	0,3871	-0,0251	-1,1235	-0,3106	-0,0951	-1,1235	-0,5553	-0,2242	-0,3085
<b>2011-2013</b>	0,2252	0,5983	0,7312	0,5983	0,1019	-0,7996	-0,6295	0,0430	-0,7996	-0,5844	-0,1098	-0,4692
	YAMB BURG ISTA			ROZH YAMB ISTA			MONT SRED ORID			TARN SHUM YAMB		
	A	B	C	A	B	C	A	B	C	A	B	C
<i>AB</i>	<i>AC</i>	<i>BC</i>	<i>AB</i>	<i>AC</i>	<i>BC</i>	<i>AB</i>	<i>AC</i>	<i>BC</i>	<i>AB</i>	<i>AC</i>	<i>BC</i>	
<b>2009-2013</b>	-0,1812	-0,0801	-0,1989	0,0055	0,1594	-0,0801	0,4530	0,0047	0,1385	-0,4377	-0,0534	-0,6112
<b>2010-2013</b>	-0,3085	-0,1304	-0,1410	-0,0251	0,0944	-0,1304	0,4180	0,0234	0,1220	-0,5462	-0,0951	-0,5553
<b>2011-2013</b>	-0,4692	-0,1213	-0,0447	0,1019	0,1069	-0,1213	0,0873	0,0056	0,0866	-0,6455	0,0430	-0,5844
	SHUM VARN BURG											
	A	B	C									
<i>AB</i>	<i>AC</i>	<i>BC</i>										
<b>2009-2013</b>	-0,7596	-0,0963	0,2477									
<b>2010-2013</b>	0,0383	-0,2242	0,4596									
<b>2011-2013</b>	0,0924	-0,1098	0,1894									

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Deformations of compression and extension of triangle sides are shown in red colour in figures 3 for the longest time period of four years 2009-2013.

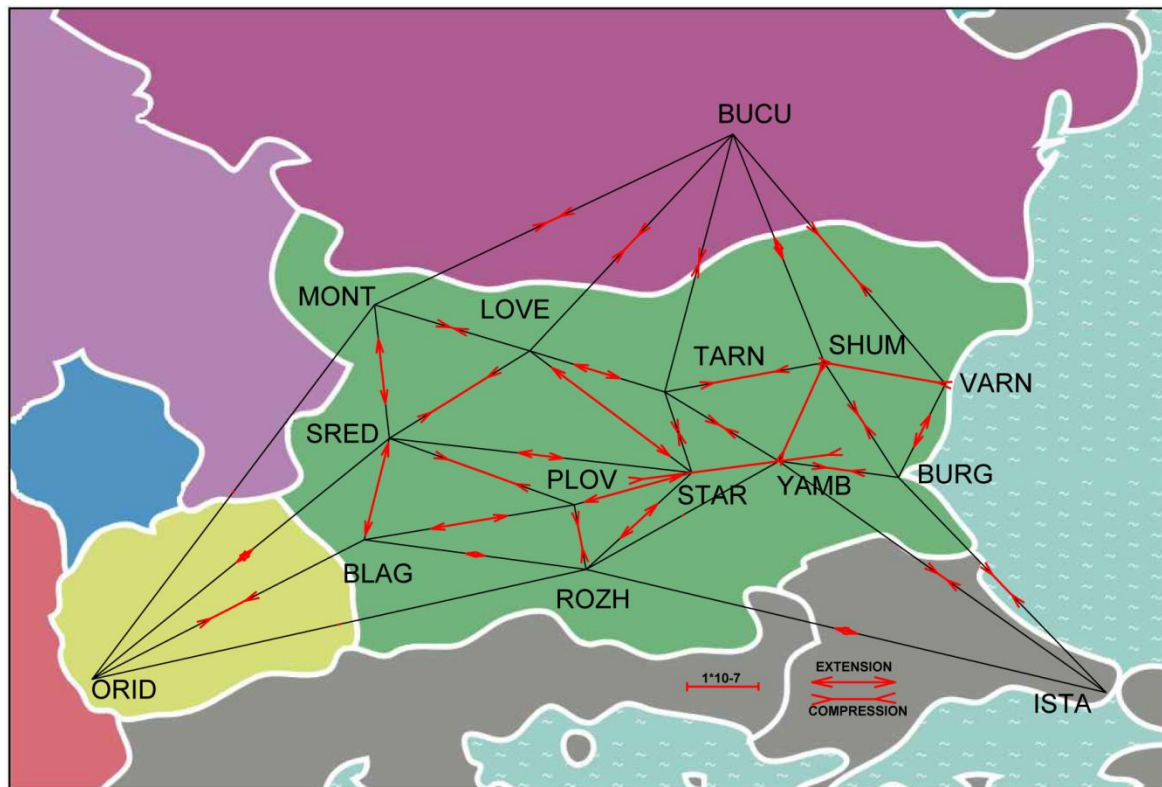


Fig. 3. Deformations of compression and extension of finite element sides for time span of four years

The smallest relative movements are obtained between station BUCU-TARN, TARN-STAR, ORID-ROZH, ORID-MONT and ROZH-YAMB, which are between  $0,005 \div 0,015 \cdot 10^{-7}$  and that corresponds to the extension of  $0,1 \div 0,2 \text{ mm}$  for all the mention sides except BUCU-TARN, which is compressed. Considering the obtained results for the west part of Bulgaria where they show an extension in direction north-south it can be concluded that there is a good agreement with the directions of estimated GPS velocities of movement from other studies [4], [8], [10], [11], [13], [22] and the results confirm the belonging of west-south Bulgaria to the Aegean extensional zone. The largest relative movements are obtained between stations STAR and LOVE – extension of amount of  $1,031 \cdot 10^{-7}$  and between stations STAR and YAMB – compression of amount of  $1,316 \cdot 10^{-7}$ . In fact these are 9,6mm, respectively - 11,3 mm linear deformations. For most of the finite element's sides there is no disagreement of the obtained deformations in all time spans studied except a few of them. The latter concerns stations SHUM, VARN, ROZH and TARN. From previous investigations [19], [20] the station VARN shows a strange behaviour. The horizontal station velocity vectors obtained from different time spans have different directions and it is suggested that the station belongs to very local microplate with some rotation [21]. For the other stations the obtained discrepancy cannot be explained and more detail study should be done. Considering the

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tectonic setting of Bulgaria [5], baseline BUCU-VARN falls in the areas between two longitudinal and one cross faults as it is seen in figure 1. Baseline BUCU-SHUM passes through the Alexandria depression and crosses the fault between two blocks (Hitrini-Kaspichan and Dobrich-Vetrino). We suggest that the location of this finite element within the longitudinal and cross faults could be a reason for this discrepancy. The largest difference of obtained results between four and two years and three and two years time span is obtained for side SHUM-VARN. It is probably a consequence from the discrepancy of sides BUCU-SHUM and BUCU-VARN. For the largest time span of four years it is obtained that the side SHUM-VARN is compressed (Figure 3) and for the 3 and 2 years time span it is extended. As the compression obtained is about 10 and 20 times larger than the extension obtained, the latter could be considered as negligible and it is assumed that this side actually is compressed. The comparison of linear deformations of the other sides of finite elements between different time spans shows a good agreement.

## 5. CONCLUSION

On the base of the obtained results, their analysis and comparison it could be generalized and suggested that in northern Bulgaria (Moesia platform) there is compression, in west Bulgaria – extension with direction north-south, in central Bulgaria (Maritsa basin) – extension and in east-south Bulgaria – compression. These are the first results obtained by the developed Finite Elements Model for the territory of Bulgaria. A denser network of stations will contribute to more precise and reliable results and validation of the obtained results from this study.

## REFERENCES

1. *Bogusz, J., A. Klos, M. Figurski, M. Jarosinski, B. Kontny.* (2013). Investigation of the reliability of local strain analysis by means of the triangle modelling. *Acta Geodyn. Geomater.*, Vol. 10, No. 3 (171), 293–305, 2013 DOI: 10.13168/AGG.2013.0029
2. *Bond, J., D. Kim, A. Chrzanowski, A. Szostak-Chrzanowski.* (2007). Development of a Fully Automated, GPS Based Monitoring System for Disaster Prevention and Emergency Preparedness: PPMS+RT. *Sensors*, 1028-1046
3. *Boucher, C., Z. Altamimi.* (2011). Memo (2008): Specifications for reference frame fixing in the analysis of a EUREF GPS campaign. Version 8: 18-05-2011.
4. *Burchfiel, B. C., R. W. King, A., Todosov, V., Kotzev, N., Durmurdzanov, T., Serafimovski, B., Nurce.* (2006). GPS results for Macedonia and its importance for the tectonics of the Southern Balkan extensional regime, *Tectonophysics*, 413, 239–248.
5. *Dabovski, Ch., I. Zagorchev.* (Edts). (2009). *Geology of Bulgaria, Volume II. Mesozoic geology.* Prof. Marin Drinov Academic publishing house, 766 pp., Zagorchev, I., Ch. Dabovski, T. Nikolov, Alpine tectonic subdivision of Bulgaria. 30-37 (in Bulg.)
6. *Deniz, I., H. Ozener.* (2010). Estimation of strain accumulation of densification network in Northern Marmara Region, Turkey. *Nat. Hazards Earth Syst. Sci.*, 10, 2135–2143, 2010 [www.nat-hazards-earth-syst-sci.net/10/2135/2010/](http://www.nat-hazards-earth-syst-sci.net/10/2135/2010/) doi:10.5194/nhess-10-2135-2010
7. *Georgiev, I., D., Dimitrov, E., Botev.* (2013). Crustal Motion Monitoring in Bulgaria and Surrounding Regions by Permanent GPS Array. *Proceedings of 7th Congress of Balkan Geophysical Society – Tirana, Albania, 7-10 October 2013*, 5 pp.

8. *Georgiev, I., D., Dimitrov, P., Briole, E., Botev.* (2011). Velocity field in Bulgaria and northern Greece from GPS campaigns spanning 1993-2008. Proceedings of 2nd INQUA-IGCP-567 International Workshop on Active Tectonics, Earthquake Geology, Archaeology and Engineering, Corinth, Greece, 54-56.
9. *Hu, Y., K. Wang, J. He, J. Klotz, G. Khazaradze.* (2004). Three-dimensional viscoelastic finite element model for postseismic deformation of the great 1960 Chile earthquake. Journal of geophysical research, Vol. 109, B12403, doi:10.1029/2004JB003163, 2004
10. *Kotzev, V., R. Nakov, Tz. Georgiev, B.C. Burchfiel, R.W. King.* (2006). Crustal motion and strain accumulation in western Bulgaria, Tectonophysics, 413, 127–145.
11. *Kotzev, V., R.W. King, B.C. Burchfiel, A. Todosov, B. Nurce, R. Nakov.* (2008). Crustal motion and strain accumulation in the South Balkan Region Inferred from GPS Measurements, in Husebye, E., ed., Earthquake monitoring and seismic hazard mitigation in Balkan countries: Proceedings of the NATO Advanced Research Workshop on Earthquake Monitoring and Seismic Hazard Mitigation in Balkan Countries, Borovetz, Bulgaria, 11–18 September 2005: NATO Science Series IV: Earth and Environmental Sciences. Volume 81, 19–43.
12. *Kovantsov, N.I.* (1981). Mathematics and romance. Publishing House “Science and Art”, Sofia, 117pp, (in Bulg., translation from Russian)
13. *Matev, K.* (2011). GPS constrains on current tectonics of southwest Bulgaria, northern Greece and Albania. Thesis, Doctor of university of Grenoble, 203pp
14. *Milev, G., Dabovski, C.* (Eds). (2006). Geodynamics of the Balkan Peninsula, Monograph, Reports on Geodesy, Warsaw University of Technology, Institute of Geodesy and Geodetic Astronomy, 647 pp.
15. *Milev, G., K. Vassileva, I. Milev, R. Pelovski, I. Andreev, G. Gerova.* (2014). Current status and activities of Satellite Ground-Based System EUPOS and EUPOS-BG. SES 2013, Proceedings of Ninth Scientific Conference “Space, Ecology, Safety”, 163-173.
16. *Milev, G., K. Vassileva, I. Milev.* (2009). Development and applications of DGNS in Bulgaria. Proceedings of the International symposium on GNSS, Space-Based and Ground-Based Augmentation Systems and Applications, 73-77.
17. *Stangl, G.* (2011). Velocity Fields from Geodesy to Geodynamics. Presentation at the Seminar, University of Vienna, 2011-04-12.
18. *Vassileva, K.* (2013). Effect of Datum Definition on Estimated Station Velocities from GPS Solutions: Case study. Proceedings of 7th Congress of Balkan Geophysical Society – Tirana, Albania, 7-10 October 2013, 5 pp.
19. *Vassileva, K.* (2013). Geodynamical study of the territory of the Balkan Peninsula from GPS solutions. Coordinates, Vol. IX, Issue 6, 37-42.
20. *Vassileva, K.* (2013). Three seasonal behaviour of Balkan Peninsula GNSS permanent stations from GPS solutions. Proceedings of the Bulgarian Academy of Sciences. Volume 66, Issue No1, 77-82.
21. *Vassileva, K., Atanasova, M.* (2014). Study of transition boundaries in Bulgaria from GPS. Tenth Anniversary Scientific Conference with international participation „Space, Ecology, Safety”, SES 2014 Sofia, 12–14 November, 2014, 10pp.
22. *Zagorchev, I.* (2011). Cenozoic Block Rotations in the Balkan Peninsula. Proceedings of 3<sup>rd</sup> International Symposium of the Black Sea region, 1-10 October, 2011, Bucharest, Romania, 220-222.

## **BIOGRAPHICAL NOTES**

**Professor Keranka Vassileva, PhD**

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Since 1988 I am dealing with *GPS* – basic principles of measuring methods, data processing, coordinate systems – datum definition, global and local, coordinate transformations, activities concerning the establishment of EUREF network and system in Bulgaria. At present my professional interest is in the field of application of *GNSS* systems for geodynamic investigations, especially for the region of Balkan Peninsula and Bulgaria, *GNSS/DGNSS* applications in Bulgaria on the base of the reference network of the European Positioning determination System (*EUPOS*) network established in the country – *BULiPOS* and application of *GNSS/BULiPOS* data in meteorology. Lecturer on Surveying and Engineering Surveying at the European Polytechnical University, Bulgaria and since July, 2012. I am Full Professor at the EPU. About 100 publications as 57 of them in international journals and proceedings.

**Professor Georgi Valev, DSc.**

**University of Architecture, Civil Engineering and Geodesy, Bulgaria**

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**Prof. G. Valev** started working as a design engineer in the Bulgarian State Company "Energoproekt" (1966 - 1969) where he took part in the planning and building of some of the biggest national projects. From 1969 to 1975 he worked in the Scientific Research Institute for Surveying and Cartography as a research associate mainly in the field of geodesy. Under his guidance and with his participation the National Station for Observing Satellites was organized. In 1975 he was appointed in UACEG. In connection with the launching and the tracing of the Bulgarian artificial satellite "Bulgaria 1300" in 1980 he has held an appointment in the Bulgarian Academy of Sciences. From 1993 to 1995 he was a Deputy Rector of UACEG. He is also a deputy chairman of the Union of the Surveyors and Land Managers in Bulgaria and a member of the International Association of Geodesy. He has given full-time lectures on the subjects: Geodesy, National Geodetic Networks, Integrated Geodesy and Optimization of Control Networks, Global Positioning System, Geodynamic investigation. From 2005 to 2012 he has been a lecturer in Shumen University. He has elaborated his own theory, algorithms and computer programs on any problems. He has more than 170 publications.

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