Computation of the Gravimetric Quasigeoid Model over Uganda Using the KTH Method

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Keywords: Geoid, Quasigeoid, Quasigeoid-to Geoid Separation, GNSS

SUMMARY

The gravimetric quasigeoid can be determined either directly by Stokes formula or indirectly by computing the geoid first and then determining the quasigeoid-to-geoid separation which is then used to determine the quasigeoid. This paper presents the computational results of the gravimetric quasigeoid model over Uganda (UGQ2014) based on the later technique. UGQ2014 was derived from the Uganda Gravimetric Geoid Model (UGG2014) which was computed by the technique of Least Squares Modification of Stokes formula with additive corrections commonly called the KTH Method. UGG2014 was derived from sparse terrestrial gravity data from the International Gravimetric Bureau, the 3 arc second SRTM ver4.1 Digital Elevation Model and the GOCE-only geopotential model GO_CONS_GCF_2_TIM_R5. The quasigeoid-to geoid separation was then computed from the Earth Gravitational Model 2008 (EGM08) complete to degree 2160 of spherical harmonics together with the global topographic model DTM2006.0 also complete to degree 2160.

Another aim of this paper is to compare the approximate and strict formulas of computing the quasigeoid-to-geoid separation and evaluate their effects on the final quasigeoid model. Using 10 GNSS/levelling data points distributed over Uganda, the RMS fit of the quasigeoid model based on the approximate formula are 27 cm and 10 cm before and after a 4-parameter fit, respectively. Similarly, the RMS fit of the model based on the strict formula are 15 cm and 6 cm, respectively. The results show the improvement to the final quasigeoid model brought about by using the strict formula to model more effectively the terrain in the vicinity of the computation point. With an accuracy of 6 cm, UGQ2014 represents significant progress towards the computation of a final gravimetric quasigeoid over Uganda which can be used with GNSS/levelling. However, with more data especially terrestrial gravity data and GNSS/levelling we anticipate that the accuracy of gravimetric quasigeoid modelling will improve in future.

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1. INTRODUCTION

For many developing countries such as Uganda, the potential of GNSS has not been fully exploited due to the absence of accurate regional gravimetric quasi-geoid models. This means that the ellipsoidal heights, which are geometrical heights cannot easily be transformed into the physically meaningful orthometric/normal heights which are required for most of the surveying/engineering applications. For many of these countries geoid determination is difficult due to the insufficient quantity and quality of terrestrial gravity data. For example, in Uganda according to the International Gravimetric Bureau (BGI) gravity database, there exist only 3624 points within the international boundaries of the country, i.e. one gravity data point for every 65 km^2. However, new advances in geoid computation techniques coupled with the availability of gravity data from GRACE and GOCE satellite missions have made it possible to determine accurate regional geoid models based on a combination of terrestrial and satellite gravity data.

The gravimetric quasigeoid can be computed either directly by using the Stokes formula/modification of the Stokes formula or indirectly by computing the gravimetric geoid first and then determining the quasigeoid-to-geoid separation, which is then used to determine the quasigeoid. In this paper we use the later technique to the determine the quasigeoid over Uganda (UGQ2014) based on the Uganda Gravimetric Geoid Model 2014 (UGG2014) which was determined using the Least Squares Modification of Stokes formula (LSMS) with additive corrections (AC), commonly called the KTH method (Sjöberg, 2003a, 2003b). The method was developed at the Royal Institute of Technology (KTH) Division of Geodesy by Sjöberg (1991, 2003a, 2003b and 2005). Compared to other methods, this method is superior because it is the only method that minimizes the expected global mean square error of the estimated geoid height. Hence, in contrast to most other methods of modifying Stokes' formula, which only strive at reducing the truncation error, the KTH method matches the errors of truncation, gravity anomaly and the Global Geopotential Model (GGM) in a least squares sense. The method has been numerically tested and successfully used in the determination of cm-level gravimetric geoid models in a number of countries with sparse terrestrial gravity data including the Baltic countries (Ellmann, 2004), Iran (Kiamehr, 2006), Tanzania (Ulotu, 2009), Central Turkey (Abbak et al.,2012) and is used in the national quasigeoid model of Sweden (Agren et al., 2009b).

The second aim of this paper is to compare the classical formula (Heiskanen and Moritz, 1967, pp.327-328) and the strict formula (Sjöberg, 2006; 2010) of computing the quasigeoid-geoid separation and determine their effect on the final quasigeoid. In the paper, the applied version of the KTH method used in this study is presented in Section 2. In Section 3, the gravity anomaly data, digital elevation model and GNSS/levelling data used are highlighted. In Section 4, the determination of UGQ2014 is presented including a comparison of the approximate and strict formulas of computing the quasigeoid-geoid separation. Finally, conclusions are presented in Section 5.

2. THE KTH METHOD

2.1 The Least Squares Estimator of the KTH method

The *Least Squares Estimator for the geoid height of the KTH method* is given by Sjöberg (2003b) as

$$N^{L,M} = \frac{R}{4\pi\gamma}\iint_{\sigma_0} S^L(\psi)\Delta g d\sigma + c\sum_{n=0}^{M}\left(Q_n^L + s_n\right)\Delta g_n^{GGM} + \delta N_{comb}^T + \delta N_{dwc} + \delta N_{tot}^a + \delta N_{tot}^e \quad (1)$$

where σ_0 is a spherical cap, R is the mean Earth radius, γ is mean normal gravity on the reference ellipsoid, $S^L(\psi)$ is the modified Stokes' function, $c = R/2\gamma$, s_n are the modification parameters, *M* is the maximum degree of the GGM, *L* is the maximum degree of modification, Q_n^L are the Molodensky truncation coefficients, Δg is the unreduced surface gravity anomaly, Δg_n^{GGM} is the Laplace surface harmonic of the gravity anomaly determined by the GGM of degree *n*. The estimator in Eq. (1) is the so-called combined estimator (Sjöberg, 2003b), which means that the truncated Stokes' formula is applied to the unreduced surface gravity anomaly after which the final geoid height is determined by adding a number of additive corrections, i.e. δN_{comb}^T - the combined topographic correction, δN_{dwc} -the downward continuation correction, δN_{tot}^a - the total atmospheric correction and δN_{tot}^e - the total ellipsoidal correction. Below we highlight the additive corrections one by one.

The *combined topographic correction* is computed as (Sjöberg 2000, 2001)

$$\delta N_{comb}^T(P) = -\frac{2\pi\mu}{\gamma}\left(H^2(P) + \frac{2}{3}\frac{H^3(P)}{R}\right) \quad (2)$$

where *P* is the computational point, *H* is the topographic height, μ is the product of the gravitational constant (*G*) and the standard topographic density (ρ), i.e. $\mu = G\rho$. Vermeer (2008) has questioned the exactness of the above formula for realistic terrains. However, as discussed in Sjöberg (2008) and (2009), Eq. (2) corresponds to the negative of the so-called *topographic potential bias* (Sjöberg, 2007), which in this case is the strict combined effect on the geoid height.

The *downward continuation (DWC) correction* can be written as (Sjöberg 2003b, 2003c)

$$\delta N_{dwc}^{L} = \delta N_{dwc}^{B,L} + \delta N_{dwc}^{te,L} \tag{3}$$

where $\delta N_{dwc}^{B,L}$ and $\delta N_{dwc}^{te,L}$ are the Bouguer shell effect and terrain effect, respectively , given by

$$\delta N_{dwc}^{B,L} = \delta N_{dwc}^{B} + c\sum_{n=2}^{\infty}\left[\left(\frac{R}{r_P}\right)^{n+1} - 1\right]\left(s_n^* + Q_n^L\right)\Delta g_n \tag{3a}$$

with

$$\delta N_{dwc}^{B} \approx \frac{H(P)\Delta g(P)}{\gamma_0} + 3\frac{H(P)}{r_P}\zeta_P - \frac{H^2(P)}{2\gamma_0}\left(\frac{\partial \Delta g(P)}{\partial H}\right) \tag{3b}$$

and

$$\delta N_{dwc}^{te,L} \approx \frac{R}{4\pi\gamma_0}\iint_{\sigma_0} S^L(\psi)\left(H_P - H_Q\right)\left(\frac{\partial \Delta g}{\partial H}\right)_Q d\sigma_Q \tag{3c}$$

In the equations above, *P* and *Q* are the point on the Earth's surface and the running point on the sphere, respectively, $r_P = R + H(P)$, ζ_P is defined by Bruns' formula, i.e. $\zeta_P = T_P/\gamma$ where T_P is the disturbing potential for point *P* and γ is the normal gravity at the normal height of point *P* and Δg_n is the Laplace harmonic in the sum in Eq. (3a) is taken from a GGM, which requires the upper limit of the sum to be set equal to or below its maximum order.

Following Sjöberg and Nahavandchi (2000) and Sjöberg (2001), the *combined atmospheric correction* can be computed as

$$\delta N_{comb}^{a} = \frac{\delta V_0^a}{\gamma} - \frac{2\pi R\rho_0}{\gamma}\sum_{n=2}^{M}\left(\frac{2}{n-1} - s_n - Q_n^L\right)H_n(P) - \frac{2\pi R\rho_0}{\gamma}\sum_{n=M+1}^{\infty}\left(\frac{2}{n-1} - \frac{n+2}{2n+1}Q_n^L\right)H_n(P) \tag{4}$$

where δV_0^a is the zero-degree term of the atmospheric potential, ρ_0 is the atmospheric density at sea level, H_n is the Laplace surface harmonic of degree *n* for the topographic height and either $s_n^* = s_n$ if $2 \le n \le M$ or $s_n^* = 0$ otherwise.

The *ellipsoidal correction to order* e^2 of the modified Stokes formula is given by Sjöberg (2004) as

$$\delta N_{total}^{e,L} = \frac{R}{2\gamma}\sum_{n=2}^{\infty}\left(\frac{2}{n-1} - s_n^* - Q_n^L\right)\left(k\Delta g_n + \frac{a}{R}\delta g_n^e\right) \quad (5)$$

where δg_n^e is the Laplace harmonics of the ellipsoidal correction to the gravity anomaly, which can be decomposed into a series as shown by Sjöberg (2003d and 2004), $k = a/R - 1$ is a scale factor and *a* is the semi-major axis of the reference ellipsoid.

3. DATA USED TO COMPUTE THE GRAVIMETRIC GEOID MODEL

3.1 Gravity Anomaly Data

The most important information concerning the terrestrial gravity anomaly data used is summarised below:

- 7839 gravity observations extracted from the BGI gravity database covering the area which lies between $3^\circ S \le \varphi \le 5^\circ N$ in latitude and $28^\circ \le \lambda \le 36^\circ$ in longitude. The distribution of the data is presented in Figure 1.

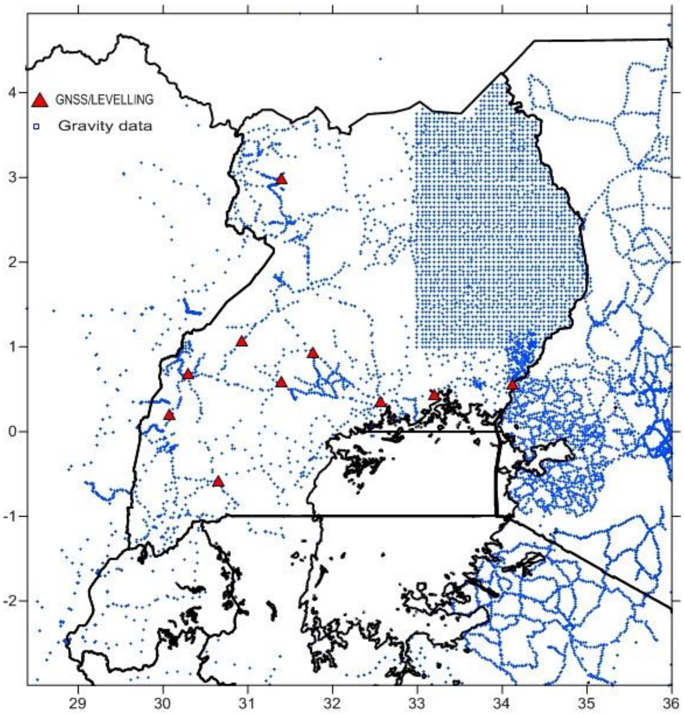

Figure 1: Location of the gravity data and GNSS/levelling benchmarks

- Outliers in the terrestrial gravity data were identified using visual inspection, direct comparison with the WGM2012 surface gravity anomalies and the use of the cross validation approach (Kiamehr, 2007; Ulotu, 2009). As a result a total of 812 gravity points representing 10.3 % of the terrestrial gravity data were identified as outliers and then removed from the gravity data.
- The Bouguer gravity anomaly was used to convert the surface gravity anomalies into reduced gravity anomalies, which are assumed to be smoother than the original surface gravity anomalies. This technique was used to overcome the challenge of interpolating unreduced gravity anomalies since the KTH method works on the full gravity anomaly without any reduction (Sjöberg, 2003b). Then the reduced gravity anomalies were interpolated to a denser grid and finally the effect of the topographic masses were removed from the Bouguer anomaly grid resulting in to free-air anomalies.
- The final grid at a resolution of $1' x 1'$ was constructed using the method of Kriging with linear variograms (Kiamehr, 2007; Ulotu, 2009).

3.2 Digital Elevation Model

The digital elevation model SRTM3 version 4.1 from the Consortium for Spatial Information of the Consultative Group of International Agricultural Research, Italy (http://www.cgiar-csi.org/data/srtm-90m-digital-elevation-database-v4-1) was used as it had the best quality in Uganda when compared to the ASTER DEM (Ssengendo et al., submitted).

3.3 Global Geopotential Models

For the computation of UGG2014, we used the GOCE-only GGM GO_CONS_GCF_2_TIM_R5 up to degree 280 since it had the lowest standard error of all GGMs evaluated with GNSS/Levelling data (Ssengendo et al., submitted). This was preferred in order to guard against correlations that may arise between the errors in the GGM and the terrestrial gravity anomalies in the case of the combined model (Ågren, 2004 and Ågren et al., 2009b).

3.4 GNSS/Levelling

Due to the absence of GNSS observations on levelled benchmarks in the country. GNSS observations using Trimble R7 GNSS receivers were carried out on 10 Fundamental Benchmarks of the Uganda vertical network marked as GNSS/levelling points in Figure 1. The heights of the 10 points are normal-orthometric heights which are based on precise levelling which was carried out in the 1960s by the British Directorate of Overseas Surveys (IGN, 2004). The ITRF08 coordinates of the 10 points were computed using the Bernese software version 5.2.

4. DETERMINATION OF THE UGANDA GRAVIMETRIC QUASIGEOID MODEL 2014

4.1 The Uganda Gravimetric Geoid Model 2014

Based on Eq. (1), UGG2014 was computed using the datasets highlighted above (Ssengendo et al., submitted). Its internal accuracy based on error propagation was estimated as 11.5 cm whereas the external accuracy based on comparison with 10 GNSS/Levelling points shown in Figure 1 was estimated as 11.6 cm and 7.4 cm before and after the 4-parameter fitting respectively (Ssengendo et al., submitted).

4.2 Determination of the Quasigeoid-Geoid Separation (QGGS)

4.2.1 Approximate Formula for the QGGS

Following Heiskanen and Moritz (1967, pp.327-328) and Hofmann-Wellenhof and Moritz (2006, pp. 324-328), the height anomaly ζ and the geoid undulation N are related by

$$\zeta - N = H - H^{*} = -\frac{\bar{g} - \bar{\gamma}}{\bar{\gamma}} H \qquad (6)$$

where H is the orthometric height, H^{*} is the normal height, $\bar{g}$ and $\bar{\gamma}$ are the mean gravity between the geoid and the Earth's surface and mean normal gravity between the reference ellipsoid and telluroid, respectively. The term $(\bar{g} - \bar{\gamma})$ is not directly available (Sjöberg, 2010) thus the QGGS can be computed by approximating this term by the simple planar Bouguer gravity anomaly (Δg_B) at the computation point (Heiskanen and Moritz, 1967, p.327) such that

$$\zeta - N \approx -\frac{\Delta g_B}{\gamma_0} H \qquad (7)$$

where $\bar{\gamma}$ in the denominator is replaced by the normal gravity for an arbitrary standard latitude (γ_0) usually 45°.

Using Eq. (7) with Δg_B obtained from the BGI gravity data, $\gamma_0 = 981000$ mGal and H extracted from the SRTM3 DEM, the QGGS over Uganda was computed. The results are presented in Table 1 (statistics) and Figure 2. As expected the QGGS is highly dependent on the elevation and hence the maximum values are observed around the Rwenzori Mountains in South-Western Uganda and Mt. Elgon in Eastern Uganda. The lowest values are observed along the Western part of the Great Rift Valley. With an average elevation of approximately 1170 m over Uganda, the QGGS lies between 0.05 m and 0.30 m with an average of 0.16 m.

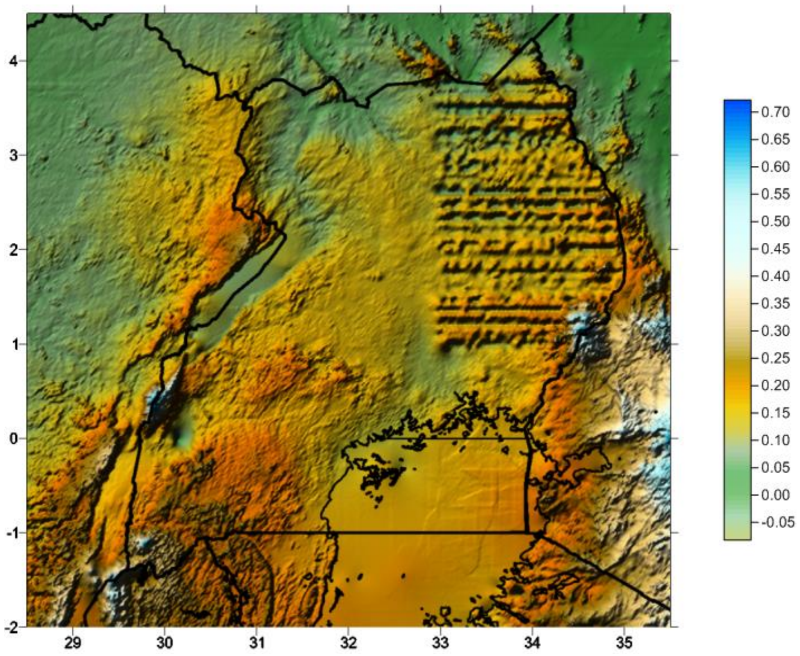

Figure 2: The QGGS over Uganda computed by Eq. (7). Unit: metre

Table 1: Statistics of the QGGS over Uganda computed using the approximate and the exact formulas (units: metres)

Formula	**Min**	**Max**	**Mean**	**Standard deviation**	**RMSE**
Approximate	-0.08	0.72	0.16	0.08	0.17
Strict	-0.05	3.35	0.17	0.19	0.25

4.2.2 A strict formula for the QGGS

According to Heiskanen and Moritz (1967, p.328), the approximate formula of Eq. (7) is only suited to giving an idea of the order of magnitude of the QGGS. Thus in order to achieve high accuracy for areas with rough terrain especially mountainous regions the QGGS must be computed by a more accurate formula. Subsequently various authors (Sjöberg, 2006; Tenzer et al., 2006; Flurry and Rummel, 2009; Sjöberg, 2010 and 2012) have presented improved practical computational formulas for the determination of the QGGS.

Following Sjöberg (2006 and 2010); see also Sjöberg and Bagherbandi (2012) and Bagherbandi and Tenzer (2013), a more accurate formula for computing the QGGS is given as

$$\zeta - N = \frac{T\left(r_P,\Omega\right)}{\gamma_Q\left(\varphi\right)} - \frac{T^*\left(r_g,\Omega\right)}{\gamma_0\left(\varphi\right)} + \frac{V_{bias}^t\left(r_P,\Omega\right)}{\bar{\gamma}\left(\Omega\right)} \qquad \text{(8a)}$$

with $T^*\left(r_g,\Omega\right)=\sum_{m=-n}^{n} T_{nm}Y_{nm}\left(\Omega\right)$ (8b)

$$V_{bias}^{t}\left(r_P,\Omega\right)=2\pi G\rho_0^t\sum_{n=0}^{n_{\max}}\sum_{m=-n}^{n}\left(H_{nm}^{2}+\frac{2}{3R}H_{nm}^{3}\right)Y_{nm}\left(\Omega\right) \quad (8c)$$

Here T is the disturbing potential at an arbitrary point $\left(r,\Omega\right)$, *R* is the Earth's mean radius, Y_{nm} are the fully normalized spherical harmonic functions of degree *n* and order *m*, T_{nm} are the fully normalized coefficients of the disturbing potential, $n_{\max}$ is the upper summation index of spherical harmonics, γ_Q is the normal gravity at the telluroid, γ_0 is the normal gravity at the reference ellipsoid, r_P is the geocentric radius of the surface point. $T^*\left(r_g,\Omega\right)$ in Eq. (8b) is the analytically continued external type harmonic series at the geoid where the true potential is not harmonic. The 3-D position is defined in the system of spherical coordinates $\left(r,\Omega\right)$, where *r* is the spherical radius and $\Omega=\left(\varphi,\lambda\right)$ is the spherical direction with the spherical latitude φ and longitude λ. $V_{bias}^{t}/\bar{\gamma}$ is the topographic bias which represents the error in the analytical downward continuation of the external gravitational potential inside the topographic masses (Sjöberg, 2007) where ρ_0^t is the mean topographic mass density and the terms $\left\{\sum_{m=-n}^{n} H_{nm}^{i}Y_{nm}\left(\Omega\right): i=1,2,3,...\right\}$ define the spherical height functions $\left\{H_n^i : i=1,2,3,....\right\}$; i.e.

$$H_n^i\left(\Omega\right)=\frac{2n+1}{4\pi}\iint_{\varphi} H^i\left(\Omega'\right)P_n\left(t\right)d\Omega'=\sum_{m=-n}^{n} H_{nm}^{i}Y_{nm}\left(\Omega\right) \quad (9)$$

where P_n is the Legendre polynomial of degree *n* with $t=\cos\psi$ i.e. the cosine of the spherical distance between spherical directions Ω and Ω'.

The practical computation of the QGGS based on Eqs. (8a), (8b) and (8c) requires three types of global models i.e. GGM, global topography and topo-density models (Sjöberg, 2006). In this study, the EGM08, complete to degree 2160 together with the global topographic model DTM2006.0 (Pavlis et al., 2006) complete to degree/order 2160 are used to compute the QGGS. The results are reported in Table 1 (statistics) and Figure 3. Overall, the QGGS varies between -0.05 m and 3.35 m with maximum values observed around the Rwenzori Mountains in South-Western Uganda and Mt. Elgon in Eastern Uganda. Compared to the approximate formula, the results of the strict formula are larger especially for the mountainous regions where the maximum values are larger by approximately 2.6 m which shows the large errors that can be introduced in the QGGS due to the use of the approximate formula.

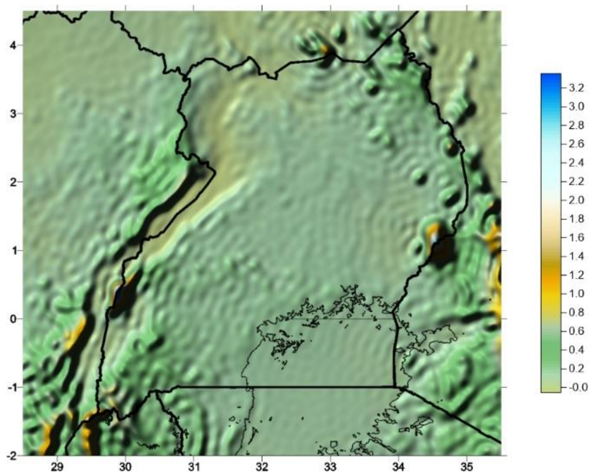

Figure 3: The QGGS over Uganda computed by the strict formula of Sjöberg (2006 & 2010). Unit: metre

4.2.3 Comparison of the approximate and strict formulas

According to Bagherbandi and Tenzer (2013), the principal difference of the approximate formula in Eq. (7) and the strict formula in Eqs. (8a),(8b) and (8c) is the consideration of the surrounding terrain in the computation of the topographic bias compared to the approximate formula where only the topographic height of the computation point is taken into account in the functional model. As shown by Sjöberg (2007) we note that although the topographic bias is a purely local phenomenon that is not affected by the terrain with only the Bouguer shell correction involved. We need the terrain for the harmonic series expansion as shown by Eq. (8c). Thus by considering the terrain, we are able to estimate the topographic bias much more accurately in the strict formula than in the approximate formula. We can see from Figure 4 that the topographic bias ranges between a minimum of 0.02 m and a maximum of 2.04 m whereby the maximum values are observed in the mountainous regions of country. With a mean of approximately 0.17 m, the topographic bias contributes about 94% to the QGGS with the remaining 6% contributed by the disturbing potential terms of Eq. (8a). This is in line with the findings of Sjöberg and Bagherbandi (2012) who have shown that globally the contribution of the topographic bias to the QGGS is approximately 90% with the remaining 10% attributed to the disturbing potential terms.

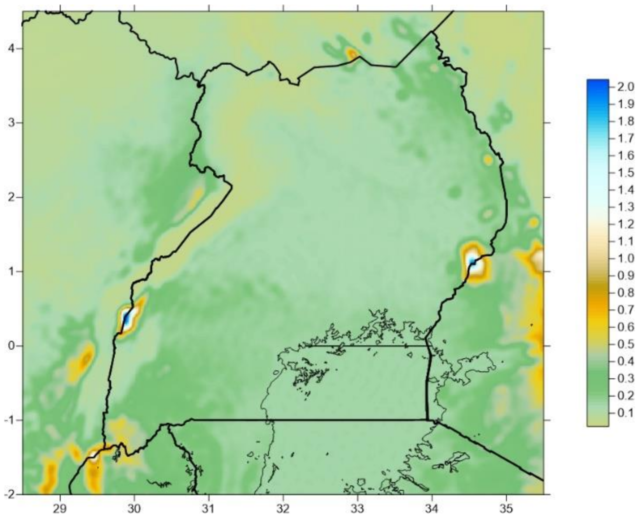

Figure 4: The topographic bias over Uganda. Unit: metre

4.2.4 Uganda Gravimetric Quasigeoid Model (UGQ2014) and its evaluation

Based on Eqs. (7) , (8a) and (8b), two gravimetric quasigeoid models were computed using Eq. (10) with *N* extracted from UGG2014.

$$\zeta = N + (\zeta - N)_i \qquad (10)$$

where i is the particular computational formula used to determine QGGS.

Subsequently the models were independently evaluated using GNSS/levelling so as to determine the best gravimetric quasigeoid for Uganda. The results of the evaluation before and after 4-parameter fitting are reported in Table 3.

Table 3: The GNSS/levelling residuals over 10 GNSS/levelling points before and after the 4-parameter fit (units: cm)

Formula		**Min**	**Max**	**Mean**	**Standard deviation**	**RMSE**
Approximate	Before	2.56	51.41	24.10	12.74	26.96
	After	-20.46	13.80	0.00	10.90	10.34
Strict	Before	-30.54	14.56	-9.29	**13.18**	**15.57**
	After	-9.96	12.91	0.00	6.65	**6.31**

From the table, it is clear that the quasigeoid model based on the strict formula fits GNSS/levelling better that the quasigeoid model based on the approximate formula i.e. in terms of root mean square error (RMSE) it is approximately better by 10 cm before the parameter fit. This may be a result of the fact that the approximate formula considers the height of the computation point only while the strict formula considers the terrain in the harmonic series expansion as shown by Eq. (8c). This leads to a much better modelling of the effect of the terrain configuration of the computational points on the QGGS hence leading to more accurate computation of the topographic bias. After the 4-parameter fitting, the quasigeoid model based on the approximate formula recovers reasonably well to within 4 cm the quasigeoid model based on the strict formula. This highlights the importance of the 4-parameter model in absorbing the systematic biases that propagate directly into the height anomalies. However, in terms of both standard deviation and RMSE, the quasigeoid model based on the strict formula still fits GNSS/levelling much better than the model based on approximate formula which again highlights the improvement to the quasi-geoid computation as a result of using the strict formula in the computation of the QGGS.

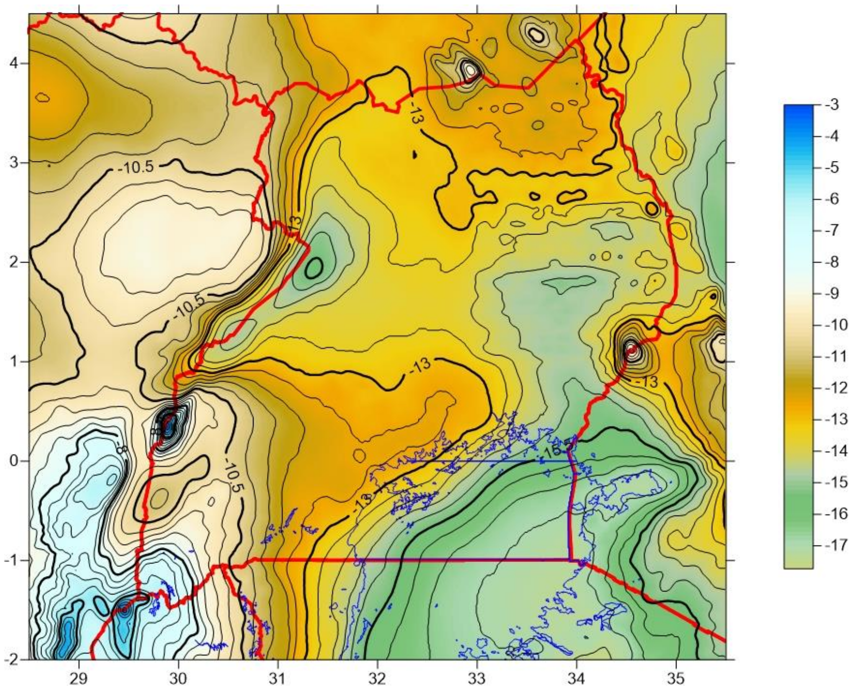

Figure 5: The Uganda Gravimetric Quasigeoid Model 2014. Unit: metre. Contour interval: 0.5 m

Based on the RMS values of 15.6 cm and 6.3 cm before and after the parameter fitting, respectively, and assuming that the standard errors of the ellipsoidal heights and the normal-orthometric heights are 2.2 cm and 1.0 cm respectively, by simple error propagation the standard error of UGQ2014 before and after fitting can be estimated as $\sqrt{(15.6)^2-(2.2)^2-(1.0)^2}=15.4$ cm and $\sqrt{(6.3)^2-(2.2)^2-(1.0)^2}=5.8$ cm. We can see that the 4-parameter model has reduced the standard error of UGQ2014 by 9.5 cm or 62% by absorbing the systematic biases.

Finally the UGQ2014 computed based on the strict formula is illustrated in Figure 5. It has the following statistics: minimum= -17.7 m, maximum = -3.0 m, mean = -12.75 m, standard deviation = 2.45 m and RMS = 12.97 m.

5. CONCLUSIONS

The main purpose of this paper was to present the computation of the gravimetric quasigeoid model UGQ2014 over Uganda. The 15.6 cm and 6.3 cm Root Mean Square Errors (RMSE) obtained by UGQ2014 before and after the 4-parameter fit respectively are very satisfactory given the quality and quantity of the terrestrial data used. If the standard errors for GNSS and levelling are taken as 2.2 cm and 1.0 cm respectively, then the propagated RMSE for the fitted gravimetric quasigeoid becomes 5.8 cm. This is encouraging given the poor quantity and quality of the terrestrial gravity anomaly data used in the computation of UGQ2014. Compared to UGG2014, the gravimetric quasigeoid appears to fit GNSS/levelling much better by approximately 1.6 cm. This is in line with the theoretical definition of normal-orthometric heights whose reference system is defined as a quasigeoid rather than the geoid.

For the comparison of the approximate and strict formulas of computing the QGGS, the strict formula leads to a better computation of the QGGS since our results show that the approximate formula introduces errors of approximately 2.6 m in the QGGS which propagate errors of up to 35 cm in the final quasigeoid. Although there is need for further studies especially with more high resolution GGMs, our results show that in regions with variable terrain especially mountainous areas the strict formula should be used in the computation of the QGGS and subsequently computation of the final quasigeoid model.

In the case of Uganda, the accuracy of UGQ2014 i.e. 5.8 cm represents significant progress since UGQ2014 is the first regional/local gravimetric quasigeoid model over Uganda. As part of future work, we anticipate that improvements in terrestrial gravity coverage as part of increased mineral exploration in the country will provide more gravity data that can be used to improve the accuracy of the gravimetric quasigeoid model. In addition more GNSS/levelling observations are needed so as to provide a much better homogeneous data set that can be used for validating and evaluating global and regional gravimetric quasigeoid models.

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