Application of Back Propagation Artificial Neural Network for Modelling Local GPS/Levelling Geoid Undulations: A Comparative Study

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Key words: artificial neural networks, geoid undulation, GPS/Levelling, back propagation, interpolation

SUMMARY

The fast development of Global Positioning System (GPS) technology provides more precise and rapid surveying in geodetic applications than the traditional terrestrial positioning techniques. Therefore, considerable savings on time, labour and cost are achieved by GPS measurements. The geometric height supplied by GPS is ellipsoidal height and it needs to be transformed to orthometric height for geodetic applications. For the transformation between ellipsoidal heights and orthometric heights, local and global geoid models generated. In the present study, a local geoid model was first generated according to interpolation methods such as polynomial, KRIG, INDW, MSHP, RBAF and LPOL from the geoid undulations obtained by using GPS/Levelling data for a study area. Subsequently, a back propagation artificial neural network (BPANN) that has been more widely applied in engineering among all other neural network models was used to generate the local geoid model of the study area with the same data. The selected interpolation methods and BPANN are evaluated, in terms of root mean square error (RMSE). In the BPANN method, RMSE was calculated as ±0.0185 m for the reference points and as ±0.0202 m for the test points. These values are smaller than the values obtained by the classical interpolation methods. Large Scale Map and Map Information Production Regulation (LSMMIPR) requires ±5 cm, accuracy level for local geoid determination, in Turkey. Therefore, it was concluded that BPANN can be used for local geoid undulation modelling as an alternative to the interpolation methods.

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1. INTRODUCTION

The fast development of artificial satellite technology has increased the importance of three–dimensional (3D) positioning and therefore, satellite geodesy. Particularly, the Global Positioning System (GPS) provides more practical, rapid, precise and continuous positioning results anywhere on the Earth in geodetic applications when compared to the traditional terrestrial positioning methods. The World Geodetic System–1984 (WGS84) is being used as the reference frame in positioning with GPS. GPS technique provides Cartesian coordinates (X, Y, Z) or the curvilinear (geodetic) coordinates (φ, λ, h) of a point on Earth as related to WGS84 ellipsoid with high precision. However, most of engineering applications often require a coordinate transformation into local coordinate systems for geodetic purposes. In the transformation of a point's horizontal coordinates, results are obtained with mm precision. However, when transforming an GPS derived ellipsoidal height into a local height system; e.g. orthometric height, a similar precision cannot be achieved due to the limited accuracy of the local geoid (Hu et al., 2002; Zaletnyik et al., 2004; Kavzoglu and Saka, 2005; Lin, 2007).

Due to the increasing use of GPS positioning techniques, a great attention has been paid to the precise determination of local/regional geoids, aiming at replacing the geometric levelling with GPS measurements. Several methods have been developed, which can be classified into two basic approaches: the geometric approach and the gravimetric approach. The geometric approach is to use the known "geoid heights" at some points, which are derived from collocated GPS derived heights and levelled heights, to interpolate the geoid heights at other points (Zhan and Yong, 1999; Chen and Luo, 2004; Lin, 2005). Geoid height refers to the difference between the WGS84 ellipsoidal height and the levelled height with respect to a local vertical datum. The gravimetric approach is to determine a geoid model using gravity measurements (Chen and Luo, 2004). In this study the geometric method that has been widely used in engineering projects for a relatively small area (Zhan and Yong, 1999) is selected.

A highly accurate geoid is useful for the transformation between GPS derived ellipsoidal heights and orthometric heights. A combination of GPS derived ellipsoidal heights and an accurate geoid model provides a new alternative method for orthometric height determination (You, 2006). Thus, labour, time, and cost savings are possible with the use of a local geoid model.

The use of ellipsoidal heights obtained by GPS for geodetic purposes requires precise determination of the local geoid. In Turkey, gravity data, topographic data and geoid heights were used to construct Turkish Geoid–2003 (TG–03) using the Least Squares Collocation (LSC) methods. For providing the current need for precise geoid heights, geoid undulations at

3’x3’ grid points on sea and land were calculated for interpolations in subsequent practical uses, on the basis of gravity anomalies in an average of 65000 stations and data on elevation from sea level in an average of 20000 stations, as well as EGM96 global geopotential model. (Erol et al., 2008).

Large Scale Map and Map Information Production Regulation (LSMMIPR) that is prepared by the Turkish Chamber of Survey and Cadastre Engineers to designate the fundamentals of practical geodetic studies, proposes four methods for transformation between ellipsoidal heights and orthometric heights. These methods are direct use of the existing geoid model, use of the existing geoid model by updating it with local GPS/Levelling points, calculation of orthometric height differences on GPS base vectors and direct use of a local geoid model based on local GPS/Levelling points without using the existing geoid model (Akiz and Yerci, 2009).

There are many articles in the literature about different interpolation methods, polynomial method and, in particular, concerning local geoid model construction using the GPS/Levelling method (Erol and Celik, 2004; Grebenitcharsky et al., 2005; Kiamehr and Sjöberg, 2006; Nunez et al., 2008). The artificial neural network (ANN) has been applied in diverse fields of geodesy and geoinformatics (Miima et al., 2001; Schuh et al., 2002; Lin, 2007; Gullu and Yilmaz, 2010; Yilmaz et al., 2010; Gullu et al., 2011). ANN was introduced as a geoid approximator by Ambrozic et al. (1999) as an alternative method (Lin, 2009). Several studies have been put for local geoid model determination by ANN in the recent years (Palancz and Volgyesi, 2003; Kavzoglu and Saka, 2005; Veronez et al., 2006; Lin, 2009).

The main objective of this study is to evaluate a back propagation artificial neural network (BPANN) for modelling local GPS/Levelling geoid undulations as an alternative method to the traditional polynomial and interpolation methods proposed in LSMMIPR. The geoid undulations that are estimated from BPANN and interpolation methods are compared to the geoid undulations based on GPS/Levelling measurements over a study area, in terms of root mean square error (RMSE) of the undulation differences.

2. THEORETICAL ASPECTS

2.1 GPS/Levelling

The ellipsoidal height (h) is the distance of a point on the Earth from the surface of the reference ellipsoid along the ellipsoidal normal. Since mathematical computations can be made directly on the ellipsoid surface, surveys performed on the physical Earth are reduced to the surface of the reference ellipsoid by a projection method.

The orthometric height (H) is the distance of a point on the Earth from the geoid along curved plumb line. The orthometric height depends on the gravity value where the point is located. The value of orthometric height is calculated by orthometric correction of the height computed by geometric levelling.

The GPS/Levelling geoid undulations are computed by (Heiskanen and Moritz, 1967):

$$N = h - H \tag{1}$$

where N denotes the geoid undulation. However, it is not possible to calculate geoid undulation for each point on the Earth, practically. Therefore, an analytical geoid surface is formed by using the points that best represent the geoid in regions with precisely determined ellipsoidal and orthometric heights. The geoid undulations for the intermediate points thus found provide great inconvenience in practice.

2.2 Interpolation Methods

The interpolation methods used in this study are chosen on the base of existing research works in respect of data density and distribution that affects the interpolation accuracy (Yang et al., 2004; Yilmaz, 2009).

In the interpolation method, either a single $N = f\,(x, y)$ function covering the entire region is used or the region is divided into parts, each of which is expressed by a function in itself, and combinations functions are used for the entire region. In the interpolation method, if N value of reference points has been determined to construct a local geoid model, geoid undulation of any intermediate point (N_C) in the constructed model is found by moving on m number of reference points around that point by using the following equation:

$$N_C = \frac{\sum_{i=1}^{m} N_i P_i}{\sum_{i=1}^{m} P_i} \tag{2}$$

where N_C indicates the geoid undulation to be calculated by interpolation, N_i indicates the geoid undulations of the reference points around the point whose geoid undulation is to be calculated by interpolation, P_i represents the weight values of the reference points and m represents the number of reference points to be used in interpolation.

2.2.1 The Kriging Method

The Kriging method (KRIG) is a geostatistical and flexible gridding method which has been extensively used in many fields such as mining, climatology and agriculture and has proved to be useful and accurate in its fields of use. KRIG uses the distance or navigation between the reference points as a function that helps surface characterisation. Thus, in order to determine the output values for each location, KRIG assigns a mathematical function to a certain number of points or all the points located within a certain area of effect. KRIG uses weighting which allows the closely located points to have a greater influence (Chaplot et al., 2006).

2.2.2 The Inverse Distance Weighting Method

The Inverse Distance Weighting method (INDW) is based on a quite simple algorithm. Therefore, it is extensively used in applications thanks to its technical appropriateness for programming. INDW is particularly used in defining continuously changing data on the same area (Yilmaz, 2009). INDW is a weighted average interpolator. With INDW, data are weighted during interpolation, so that the influence of one point, relative to another, declines with distance from the grid point. Weighting is assigned to data through the use of a weighting power, which controls how the weighting factors drop off as distance from the grid point increases (Yang et al., 2004).

2.2.3 The Modified Shepard's Method

The Modified Shepard's method (MSHP) uses an inverse distance weighted least squares method. The surface generated with MSHP interpolates each scatter point and is influenced most strongly between scatter points by the points closest to the point being interpolated. MSHP has been used widely because of its simplicity (Yilmaz, 2009).

2.2.4 The Radial Basis Function Method

The Radial Basis Function method (RBAF) is the name given to a large family of exact interpolators. In many ways the RBAF methods applied are similar to those used in geostatistical interpolation, but without the benefit of prior analysis of variograms. On the other hand they do not make any assumptions regarding the input data points and provide excellent interpolators for a wide range of data. For the earth sciences, generally, the multi-quadric function has been found to be more effective (Smith et al., 2007).

2.2.5 The Local Polynomial Method

The Local Polynomial Method (LPOL) creates a surface which is optimized for a neighbourhood. The reference points in a neighbourhood can be weighted by their distances from the prediction location with inverse distance weighting (Kidner et al., 1997).

2.2.6 The Polynomial Method

The Polynomial method is one of the methods most commonly used to express the study area by a single function. This method generates constant coefficients by the reference points whose values are known to form the geoid surface model and calculates the unknown values for new points using these constant coefficients. The high order polynomial equation used to form a local geoid model in the polynomial method that the plane coordinates of the reference points are used as variables.

2.3 Artificial Neural Networks

ANN is a highly simplified model of decision-making processes of a human brain and is formed by artificial neurons. The input information of the neuron is manipulated by means of synaptic weights that are adjusted during a training process. After the training procedure an activation function is applied to all neurons for generating the output information (Leandro and Santos, 2007). ANN was proposed as multilayer perceptron (MLP) model in this study because of its simple implementation among several kinds of ANNs. MLP consists of one input layer with K inputs, one hidden layer with q units and one output layer with n outputs. The output of the model (y) with a single output neuron can be represented by:

$$\mathrm{y} = \mathrm{f}\left(\sum_{j=1}^{q} W_j f\left(\sum_{l=1}^{K} w_{j,l} x_l\right)\right) \tag{3}$$

where W is the weight between the hidden layer and the output layer, w is the weight between the input layer and the hidden layer, x is the input parameter. The activation function that is used for ANN is the sigmoid function, represented by:

$$\mathrm{f}\,(\mathrm{z}) = \frac{1}{1+e^{-z}} \tag{4}$$

where z is the input information of the neuron and $f(z)$ Є [0,1]. The proposed ANN for estimating the geoid undulations is trained using the back propagation algorithm that has well–known ability as function approximators (Pandya and Macy, 1995).

2.3.1 Back Propagation Artificial Neural Network

Back propagation algorithm was first defined by Werbos (1974) and later improved and recommended by Rumelhart et al. (1986). BPANN is commonly used in many fields, particularly in engineering because of its high learning capacity and simple algorithm. This algorithm aims to reduce errors backwards, from input to output.

BPANN is a feed forward and supervised learning network. In general, BPANN consists of an input layer, one or two intermediate hidden layers, and an output layer. Each layer contains different number of neurons in accordance with the problem in question (Zhang et al., 1998). According to Bishop (1995), a network with one hidden layer using a sigmoid activation function can approximate any continuous function given a sufficient number of hidden neurons. Figure 1 shows the architecture of BPANN with a single hidden layer. The delta rule based on squared error minimization is used for BPANN training procedure (Haykin, 1999).

In the training process, the weights between the hidden layer and the output layer are adjusted according to the data set that is composed of the known input and output parameters. This iterative procedure adjusted the weights in order to decrease the residuals (difference between the estimated output and the actual output) of the output of the neural network. The training procedure consists of two main steps: Feed-forward and back-propagation.

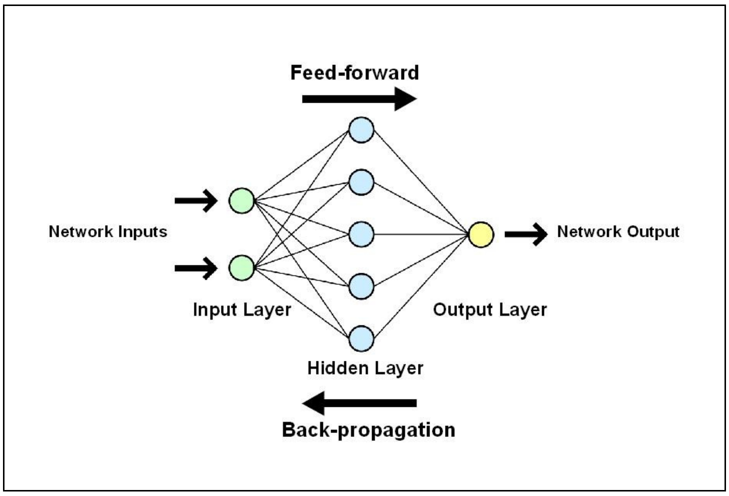

Figure 1. The BPANN architecture

The mean square error (MSE) can be used as a neural network performance indicator in the training process. For a given set of *K* inputs, MSE is defined by:

$$\text{MSE} = \sum_{i=1}^{K} (y_{i\,actual} - y_{i\,estimated})^2 \,/\, \text{K}^2 \tag{5}$$

where y_{actual} denotes the actual (target) output value and $y_{estimated}$ denotes the estimated (neural) output value. BPANN is a self-adapted method. It can reduce the model error and its fitting accuracy is high. The details about the training procedure of BPANN can be found in numerous sources including Fausett (1994), Bishop (1995) Ripley (1996) and Haykin (1999).

3. STUDY AREA, SOURCE DATA AND COMPARATİVE STUDY

In this study, the estimating of the geoid undulations were performed over a study area that is located in the province of Afyonkarahisar (Turkey) within the geographical boundaries: $30^0\ 23'\ \text{N} \leq \varphi \leq 30^0\ 34'\ \text{N}$ and $38^0\ 26'\ \text{E} \leq \lambda \leq 38^0\ 42'\ \text{E}$. The evaluating tests of the present study refer to 38 homogenous distributes points in the study area. The geodetic coordinates of the points were determined by the static GPS surveying method using 4 points in the Turkish Fundamental National GPS Network (TNFGN) close to the study area. The orthometric heights of the points were calculated by the geometric levelling method using a digital level from two points whose orthometric heights were already known.

14 points were selected as reference points in the study area and the remaining 24 points were used as test points. The reference points cover the study area from the outside and the test points were selected within the network formed by the reference points. By Eq. (1), geoid

undulations of the points were calculated using their ellipsoidal heights and orthometric heights. Figure 2 shows the 3D model obtained from the geoid undulations of the reference points and Figure 3 presents the contour map of the study area.

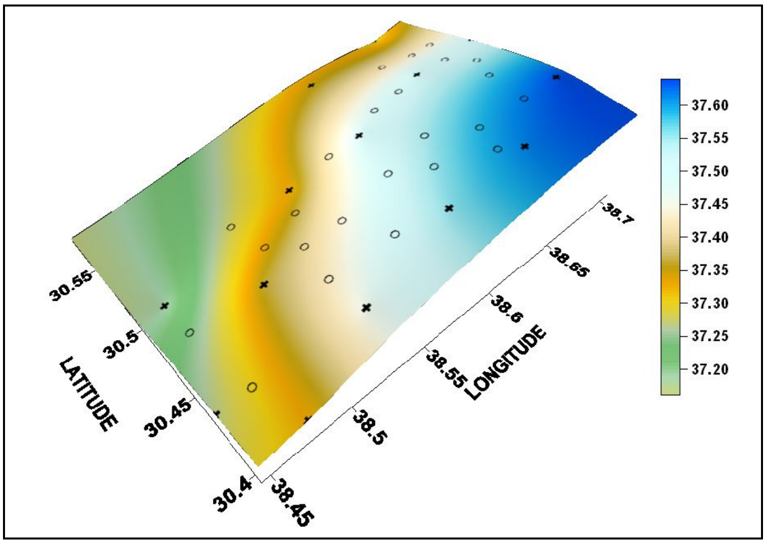

Figure 2. 3D model of the study area (+ : reference points, o : test points)

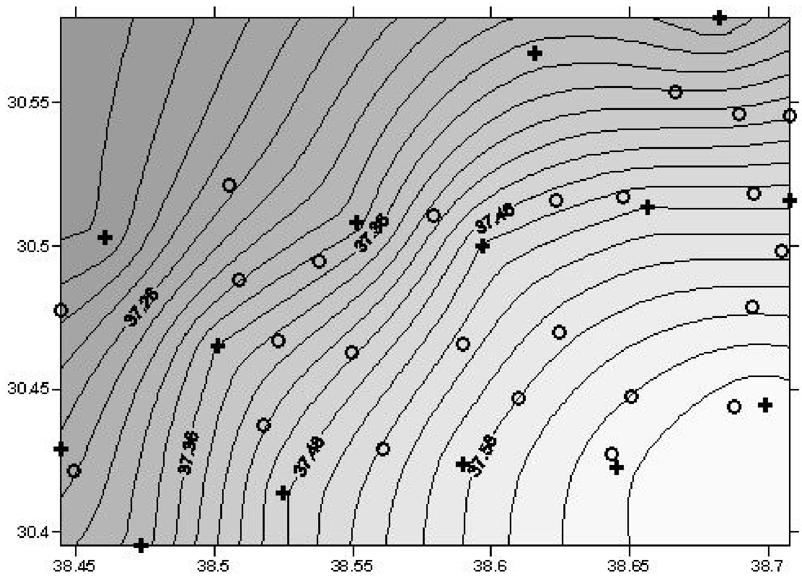

Figure 3. Contour map of the study area (+ : reference points, o : test points)

Using the geodetic coordinates and geoid undulations of the reference points, 3D models of the study area were generated, on the basis of KRIG, INDW, MSHP, RBAF and LPOL methods, by Surfer 8.0 surface modelling program. From this constructed model, the undulation differences were computed of both the reference and test points used in the model from the GPS/Levelling based geoid undulation. In accordance with the concerns specified in the LSMMIPR, with the use of the projection coordinates and geoid undulations of the points, the following equation of the third degree,

$$N = A_0 + A_1Y + A_2X + A_3Y^2 + A_4YX + A_5X^2 + A_6Y^2 + A_7Y^2X + A_8YX^2 + A_9X^3 \qquad (6)$$

was used for adjustment of the correction equations generated for each of the reference points using the least squares method and to calculate their polynomial coefficients. Using these coefficients, the differences were computed of the reference and test points from the GPS/Levelling based geoid undulation. The statistical values of these computed undulation differences are given in Table 1 for the reference points and in Table 2 for the test points.

Table 1. Statistical values of computed undulation differences for the reference points.

	INTERPOLATION					
	KRIG	INDW	MSHP	RBAF	LPOL	POLYNOMIAL
Min (m)	-0.0528	-0.0427	-0.0381	-0.0389	-0.0485	-0.0410
Max (m)	0.0559	0.0447	0.0464	0.0443	0.0337	0.0397
Mean (m)	-0.0009	-0.0004	0.0034	0.0024	-0.0067	0.0023
RMSE (m)	0.0388	0.0244	0.0242	0.0239	0.0245	0.0236

Table 2 Statistical values of computed undulation differences for the test points.

	INTERPOLATION					
	KRIG	INDW	MSHP	RBAF	LPOL	POLYNOMIAL
Min (m)	-0.0431	-0.0427	-0.0424	-0.0435	-0.0516	-0.0466
Max (m)	0.0967	0.0450	0.0415	0.0402	0.0312	0.0470
Mean (m)	0.0195	0.0077	0.0099	0.0089	-0.0001	-0.0020
RMSE (m)	0.0489	0.0269	0.0266	0.0261	0.0246	0.0243

ANN model with a hidden layer was used to compute the geoid undulations using BPANN method. The input layer consists of three neurons; i.e., geographical latitude (φ), geographical longitude (λ) and ellipsoidal height (h). Geoid undulation (N) is selected as the output layer's single neuron. After a trial-and-error strategy the number of neurons in the hidden layer was selected as 20 for minimum RMSE. Thus, the optimum BPANN structure [3:20:1] was found as a result of different trials and the network was trained by a software program developed in MATLAB. The developed program allows to dynamically changing the parameters of a learning algorithm, to monitor error values and weight changes, and to generate digital data and graphs that show whether learning is sufficient.

The evaluation of BPANN is focused on the determination of the differences between the known geoid undulation of the point and the geoid undulation of the point estimated by BPANN, using the equation below:

$$\Delta N = N_{known} - N_{estimated} \tag{7}$$

where ΔN is the geoid undulation residual, N_{known} is the known geoid undulation of the point through GPS/Levelling and $N_{estimated}$ is the geoid undulation based on BPANN method.

The geoid undulation residuals are investigated by RMSE value because RMSEs are sensitive to even small errors, which is good for comparing small differences between estimated and known discharges on models. RMSE is defined by:

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\Delta N)^2} \tag{8}$$

The statistical values of these undulation residuals are given in the Table 3 for reference points and in the Table 4 for test points.

Table 3. Statistical values computed for the reference points using BPANN method

	ITERATION NUMBERS					
	50000	100000	150000	200000	250000	300000
Min (m)	-0.0427	-0.0389	-0.0383	-0.0417	-0.0368	-0.0468
Max (m)	0.0447	0.0443	0.0427	0.0372	0.0403	0.0287
Mean (m)	-0.0004	0.0024	0.0026	-0.0010	0.0033	-0.0063
MSE (m)	0.0004	0.0004	0.0004	0.0004	0.0003	0.0004
RMSE (m)	0.0204	0.0199	0.0196	0.0191	0.0185	0.0190

Table 4. Statistical values computed for the test points using BPANN method

	ITERATION NUMBERS					
	50000	10000	150000	200000	250000	300000
Min (m)	-0.0427	-0.0435	-0.0430	-0.0459	-0.0416	-0.0499
Max (m)	0.0450	0.0402	0.0411	0.0376	0.0426	0.0324
Mean (m)	0.0077	0.0089	0.0094	0.0060	0.0010	0.0015
RMSE (m)	0.0229	0.0221	0.0222	0.0211	0.0202	0.0204

4. RESULTS AND CONCLUSIONS

Ellipsoidal heights obtained by GPS are geometrical heights and cannot be used in practical applications as they have no physical meanings. It is necessary to use the orthometric heights in most of the practical geodetic applications. Determination of orthometric heights is rather time-consuming, demanding, and costly. Therefore, studies on using ellipsoidal heights in place of orthometric heights have concentrated on calculating and modelling geoid undulations obtained by Eq. (1), or generating a local GPS/Levelling geoid. In recent years, the number of studies has been increasing on the usability of ANN method in modelling of a local GPS/Levelling geoid to replace the interpolation methods and the polynomial method used to represent irregular surfaces.

This study was carried out to generate a local GPS/Levelling geoid using BPANN method as an alternative to the fundamentals of construction local GPS/Levelling geoids in accordance with LSMMIPR conducted in Turkey.

The test results in this study indicate that:

(1) The geoid undulation differences in Table 1 and Table 2 reveals that the results of the KRIG method are greater; that the results of the INDW, MSHP, RBAF, and LPOL methods are similar; and that the results obtained by the polynomial method specified in the LSMMIPR are better when compared to the interpolation methods.

(2) The results based on BPANN presented in Table 3 and Table 4 point out that RMSE slightly decreases with increasing number of iterations; yet, the lowest MSE and RMSE values are obtained when the iteration number is 250000 and 300000.

(3) The reference points' results in Table 1 and Table 3, the test points' results in Table 2 and Table 4 show that BPANN method's undulation estimation accuracy is better than the other interpolation and polynomial methods, in terms of RMSE.

(4) Using BPANN method, this study found a RMSE of ±1.85 cm for reference points and a RMSE of ±2.02 cm for test points. These values are below the fundamental limit value of ±5 cm, which is required for fitting the generated local GPS/Levelling geoid, as specified in LSMMIPR.

(5) In generating a local geoid model, a bivariate polynomial should be used so that continuity is achieved in case the study area is later expanded. This is unfavourable as it will increase the degree of the polynomial used in modelling. It is reported that RMSE increases particularly after eighth degree with increasing polynomial degree and that the best results are obtained between the fifth and eighth degrees (Yilmaz and Arslan, 2005).

(6) When solving geodetic problems, the data are assumed to have a normal distribution. In BPANN method, however, no assumptions are generally made about the data distribution and the aim is to decrease errors backward from the output value, which constitutes an advantage.

Therefore, BPANN method is easily programmable with decreased and increased number of reference points when generating a local GPS/Levelling geoid and it performs a flexible modelling. Also, BPANN method is open to updating which could be accepted as an important advantage. Thus, it is believed that BPANN method is more convenient for generating local GPS/Levelling geoid, when compared to other methods.

From the results of this study we can conclude that BPANN based geoid undulation accuracies are more accurate than interpolation methods based geoid undulations so BPANN can be trained to predict for geoid undulations modelling as a surface approximator. Furthermore, for the future geoid modelling projects BPANN should provide an alternative to current approach techniques or should be used to refine the geoid surface's approximation such as a trend surface for other geoid modelling techniques.

BPANN method can be declared as better than interpolation methods in practice for modelling local geoid on account of effectiveness and it can be stated that combining the BPANN as a trend surface approximator with other techniques like least squares collocation will provide a significant refinement for the modelling local geoid undulations, in terms of accuracy for future studies.

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