The Geoid Geopotential Value for Unification of Vertical Datums

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SUMMARY

Vertical datums are typically realised from sea level observations at a single or multiple tide gauges (TG). Unification of these datums can be undertaken within geopotential space for mapping agencies throughout the world. This requires knowledge of the fundamental geodetic parameter W_0 which defines the Gauss-Listing geoid. In this study, the UK, French and German datums are unified on the basis of their geopotential values. Sea level information at TG sites is used to establish the mean sea level (MSL) which is connected to the Earth's centre of mass by means of GPS and precise levelling. This yields geodetic coordinates at MSL which are reduced to the geoid using a mean dynamic topography (MDT) model. By utilising the geoid coordinates within a high resolution and precision Earth gravity field model (i.e. EGM2008) yields the geopotential value which defines the geoid at the TG sites. This can be used to connect vertical datums between different regions.

Alternatively, W_0 can be estimated globally using an altimetry-based MSL, a global geopotential model and MDT and is used to connect regional datums to the world height system.

Our results show that, at the current accuracy of sea level measurements, GPS observations and processing strategies, unification of vertical datums is mainly influenced by the accuracy of the geopotential model and MDT. The German datum has the largest offset -1.30 m^2s^{-2} from the global geopotential value estimated as W_0=62636854.29±0.5 m^2s^{-2} while the datum in Stornoway has the smallest offset -0.45 m^2s^{-2}.

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1. INTRODUCTION

Many local vertical datums (LVD) exist around the world, which differ between countries and regions according to the reference surface and height system adopted in these regions. Most of these datums are based on averaging sea level measurements of a single tide gauge (TG) (e.g. UK and France) or from multiple TGs (e.g. New Zealand and Australia). The current development of GPS and other space geodetic techniques have motivated unification of vertical datums into a single global vertical datum from which a world height system can be defined. Practically, this involves determination of the geopotential value (W_0) of the level surface which globally represents the ideal ocean surface or the so-called Gauss-Listing geoid. Here we define the local or regional datum by W_0 of the surface which best fits mean sea level in a defined region, while the global vertical datum is represented by W_0 obtained for the surface which best fits the ocean surface globally. These definitions provide consistency between regional and global vertical datums and also enable monitoring of changes in these datums due, for example, to sea level change and mass redistribution. W_0 can be determined by constructing points on the geoid and estimating the geopotential values at these points using a high resolution and precision global Earth gravitational model (GGM). This requires accuracy in positioning and gravity field determination.

For regional datums, TG data together with co-located GPS measurements and levelling information can be used to establish mean sea level (MSL) at the TG locations, while globally a gridded MSL model is constructed using sea surface heights supplied by satellite altimetry. Both MSLs should be reduced to the geoid via a measure of mean dynamic topography by utilising an oceanographic MDT model. Ardalan et al (2002) attempted to unify vertical datums for countries around the Baltic Sea using data from TG, GPS, levelling and various geoid solutions with no consideration of MDT. Their study utilised the ellipsoidal harmonic coefficients of EGM96 (Lemoine et al. 1998) to degree/order 360/360. Their overall estimation of W_0 for the Baltic region was $W_0 = 62636855.75 \pm 0.21\ m^2s^{-2}$.

Global estimate of W_0 has been the subject of several studies (e.g. Bursa et al. 1999a; Burša et al. 1999b; Burša et al. 2007) which used data from satellite altimetry over the oceans. These studies concluded that the effect of degree n>120 of GGM on W_0 is negligible and also the tide system of the gravity field has no influence on W_0 determination. Their estimation of the geoid geopotential $W_0 = 62636856.0 \pm 0.5\ m^2s^{-2}$ is adopted by the International Astronomical Union (IAU). Sanchez

(2007), determined W_0 from different MSL models and different GGMs, showed that the choice of MSL and GGM is unimportant for estimating W_0 while the latitude domain of the altimetric MSL plays a major role. In the Sanchez (2007) study, the estimated W_0 differs by 2.5 m^2s^{-2} from the IAU adopted value. In a more recent study, Dayoub et al. (2011) researched the previous studies using various methods and datasets. That study confirmed the above conclusions but reported and recommended a different value $W_0 = 62636854.2 \pm 0.5$ m^2s^{-2} and established that the dependency of W_0 on the latitude domain is merely due to MDT.

This study is split into two parts: (i) Regional: This uses 12 TGs from the UK, France and Germany. These TGs are referenced to the Earth centre of mass by means of continuously operating GPS (CGPS), see Fig. 1. The TG and CGPS datums are connected by precise levelling. Gravity potential values are determined at each TG location by means of the Earth gravitational model EGM2008 (Pavlis et al. 2008) to its full spectral resolution. Consistencies between geopotential values are first sought by investigating the role of MDT in this issue. Based on the results of this part, W_0 for the considered regions is defined.

(ii) Global: An altimetry-based MSL, namely CLS01(Hernandez and Schaeffer 2001), is used to provide a globally gridded MSL model. This model is reduced to the geoid by means of an altimetry-independent MDT model (i.e. ECCO-2). Gravity potential values are then estimated at each grid point by means of EGM2008 and averaged globally to obtain W_0. In a later step the offsets between the regional datums and the global datum is concluded.

2. METHODS

The gravity potential can be estimated at any point on the Earth's surface using the following formula (e.g., Heiskanen and Moritz, 1967, Hofmann-Wellenhof and Moritz, 2006).

$$W(r,\theta,\lambda) = V(r,\theta,\lambda) + \Phi(r,\theta) \qquad (1)$$

In Eq. (1) V is the gravitational potential which can be computed by utilising the spherical harmonic coefficients of GGM to degree/order n/m

$$V(r,\theta,\lambda) = \frac{GM}{r}\left(1 + \sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(\frac{a_E}{r}\right)^n \bar{P}_{nm}(\cos\theta)(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda)\right) \qquad (2)$$

and Φ is the centrifugal potential

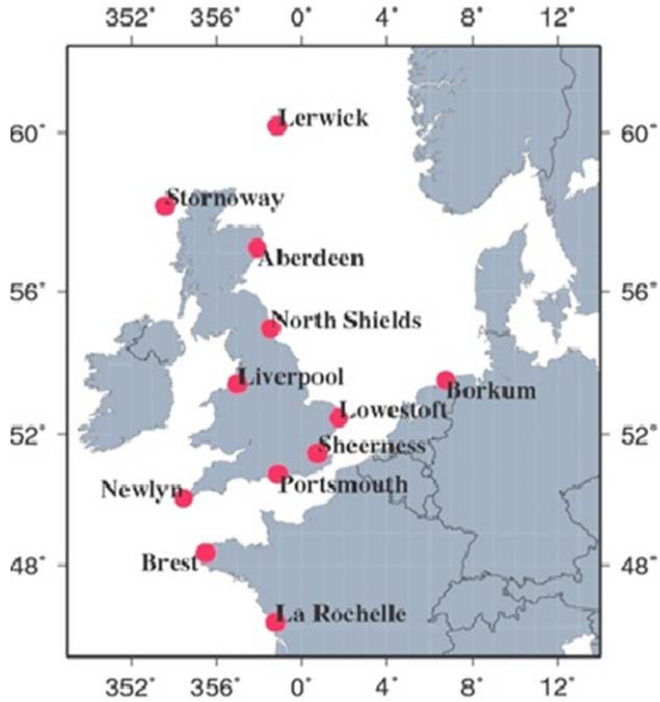

Figure 1: TG & CGPS stations for the UK, France and Germany.

$$\Phi(r,\theta) = \frac{1}{2}\omega^2 r^2 \sin^2\theta \qquad (3)$$

Also in Eq. (1-2) r, θ and λ are the geocentric radius, the co-latitude and longitude respectively; $\bar{P}_{nm}$ is the fully normalised associated Legendre functions of degree/order n/m; C_{nm} and S_{nm} the spherical harmonic coefficients of GGM; a_E the radius of the Earth's ellipsoid; GM the product of the gravitational constant and the total mass of the Earth and ω the angular velocity of the Earth. $GM = (398600441.5 \pm 0.8)\text{x}10^6\ m^3 sec^{-2}$ and $\omega = 7292115 \text{ x } 10^{-11}\ rad\ s^{-1}$are adopted for this study from the IERS 2003 conventions (McCarthy and Petit. 2004).

Regionally, MSL is constructed using TG records, co-located CGPS observations and levelling information, while globally, satellite altimetry provides a direct measure of sea surface heights which can be used to construct MSL. Both MSLs are reduced to the geoid by means of MDT as Fig. 2 shows. To estimate the gravity potentials at each geoidal point, the spherical harmonic coefficients of GGM are used in Eq. (1-3). The mean of the potential values at TG locations for each region represents W_0 for that region (W_{0LVD}), while the global average of values at the global grids gives W_0 defines the geoid globally.

The offsets between LVDs and the global geoid within geopotential space is given by (e.g. Burša et al. 1999b)

$$\delta W_0 = W_0 - W_{0LVD} \quad (4)$$

These offsets can be used to unify vertical datums of the regions considered within the global vertical datum.

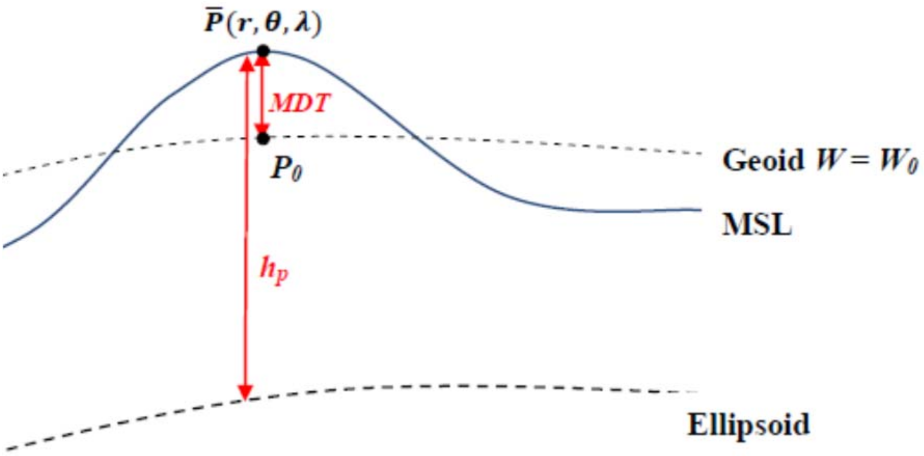

Figure 2: The geoid from MSL & MDT.

3. REGIONAL DATUM

In this study, 12 TG sites are used with two in France, one in Germany and nine in the UK, see Fig.1. Seven of the UK TGs belong to the UK mainland (i.e. England, Scotland and Wales) while the other two (Lerwick and Stornoway) belong to different datums. The TG records are obtained from the Permanent Service of the Mean Sea Level (PSMSL) in the Revised Reference Format (RLR). All records have a minimum of 30 years of data and some span a century or more. These long records allow determination of a rigorous MSL as they enable elimination of all tidal periodic constituents including the longest period of 18.6 years caused by the precession and nutation of the Sun and Moon (Ardalan and Safari. 2005). We utilised co-located CGPS data at TGs between 1999.0 and 2009.5 from the BIGF sponsored by the National Environment Research Council for the UK data and from EUREF/IGS for the French and German data. The

GPS data were processed using the GIPSY OASIS 5.0 Software in the precise point positioning (PPP) mode (Zumberge et al., 1997) in 24 hour sessions. Reprocessed non-fiducial JPL orbits were used and Earth rotation parameters and satellite clocks were held fixed. GPS corrections such as tropospheric delay, antenna phase centre, Earth tides and ocean tide loading were treated carefully. The non-fiducial coordinates were transformed to the IGS realisation of ITRF2005. The daily coordinates were averaged to obtain mean yearly coordinates for each site. This should provide *mm* level accuracy in estimating the height component of the GPS coordinates.

TG and CGPS data together were used with precise levelling information to construct MSL connected to the Earth centre of mass at each site. This was then reduced to the geoid by means of the MDT obtained from the Proudman Oceanographic Laboratory Coastal Ocean Modelling System (POLCOMS) MDT model (Holt and James, 2001). POLCOMS provides MDT values for the regions between 40º-65º *N* and 20º*W*-13º*E* with longitudinal and latitudinal resolutions 1/6º and 1/9º respectively, see Fig 3. This model is oceanographic based on the three-dimensional baroclinic model POL3DB (Holt and James, 2001). Having obtained the geodetic coordinates of the geoid at TGs, the geopotential value at each location was estimated using Eq. (1-3) by employing the Earth gravitational model EGM2008 to degree/order 2160/2160. We also estimated the gravity potentials at MSL for the same locations to show the effects of MDT on the consistency between solutions. The use of EGM2008 to degree/order 2160/2160 instead of EGM96 to degree/order 360/360 reduces the omission errors in the solution as reported in Dayoub et al (2010). However, the remaining parts of Eq. (2) for n>2160 will still produce errors that can be important in the regional scale solution.

Results in Table 1 show that using MDT has reduced the difference between the minimum and maximum W_0 values by almost 0.5 m^2s^{-2} and also reduced the scatter between the results with 11 out of 12 values are within a 1 m^2s^{-2} band. The use of MDT has also reduced the standard errors which reflect better consistencies between LVDs.

4. GLOBAL DATUM

To estimate W_0 globally, the mean sea-surface model MSSCLS01 (for brevity CLS01) (Hernandez and Schaeffer 2001) was used. This model is an altimetry-based MSL established as a continuous global surface for latitudes between 82° N and 80° S and derived from TOPEX/Poseidon (1993-1999), the geodetic phase of ERS-1 (1994- 1995), ERS-1/2 (1993-1999) and GEOSAT (1987-1988). In addition, this model uses the geoid heights from EGM96 over land with a proper smoothing performed to connect both models (i.e. EGM96 and CLS01). For this study, only data over the oceans from were used on a 1ºx1º global grid. To reduce MSL to the geoid, the Estimating the Circulation and Climate of the Ocean, phase II (ECCO-2) MDT model (Menemenlis, Campin et al. 2008) was used. ECCO-2 is an oceanographic model that covers latitudes 78° N to 78° S and constructed jointly by Jet Propulsion Laboratory (JPL), the

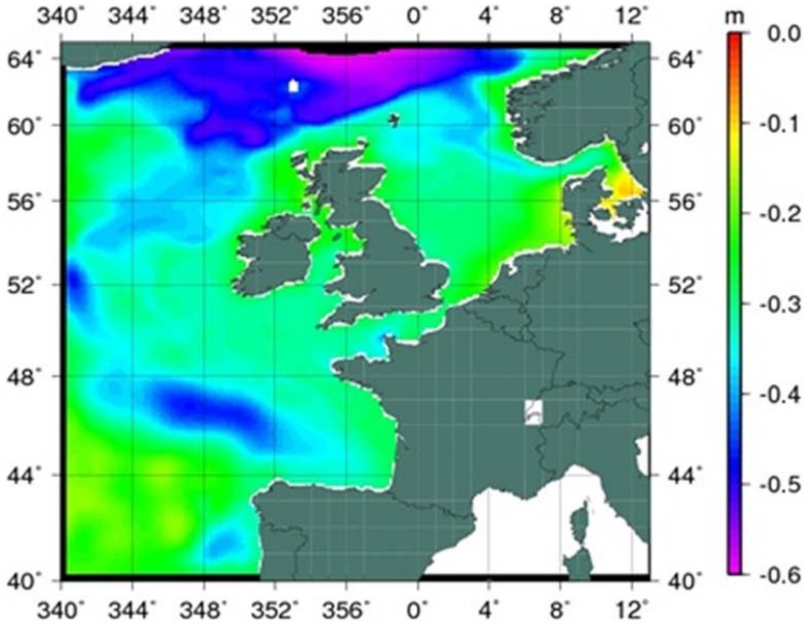

Figure 3: The POLCOMS model.

Table1: W_0 at MSL and the geoid in the UK, France and Germany.

Station	**Datum**	**GPS Data span**	**W_0-62636850 (m^2s^{-2})**	
			EGM2008 only	**EGM2008+ POLCOMS**
Aberdeen	UK mainland	1999.0-2009.5	6.95	4.48
Liverpool		1999.0-2009.5	8.64	5.44
Lowestoft		1999.0-2009.5	8.15	5.30
Newlyn		1999.0-2009.5	8.61	5.72
North Shields		1999.0-2009.5	7.48	4.85
Portsmouth		2001.0-2009.5	7.61	4.66
Sheerness		1999.0-2009.5	7.51	4.74
Lerwick	Lerwick	2005.6-2009.5	8.28	5.06
Stornoway	Stornoway	2005.7-2009.5	7.13	4.74
Brest	France	1999.0-2009.5	8.35	5.39
La Rochelle		2002.0-2007.0	8.33	5.55
Borkum	Germany	2001.0-2007.0	7.57	5.59
Average			7.88±0.33	5.13±0.24

Massachusetts Institute of Technology (MIT) and the Scripps Institution of Oceanography (SIO). EGM2008 limited to degree/order 360/360 in the mean tide system was used to determine the gravity potential values at each grid point of the model by means of Eq. (1-3) before and after removing MDT. We limited the used model to latitudes between 70º N and 70º S because MDT from ECCO-2 for latitudes 72.5° -78° N/S is supplied as one default value.

The resulting geopotential values were averaged globally using cos θ as a weighting function. Table 2 demonstrates that the use of MDT at the adopted latitude domains did not change W_0 significantly. However, it reduced the standard errors by a factor of three. These standard errors reflect only the internal accuracy of the solution and do not count for the 2-3 *cm* altimeter calibration errors (Bursa et al. 2007). This suggests the use of ±0.5 m^2s^{-2} is a safe estimate of the standard errors as recommended by Bursa et al. (2007). Thus, the resulting global geopotential value estimated with the use of ECCO-2 is W_0=62636854.29±0.5 m^2s^{-2}. This value can be used in defining the world height system.

Table 2: W_0 from CLS01, ECCO-2 and EGM2008.

Model	Degree	W_0 (m^2s^{-2}) EGM2008 only	W_0 (m^2s^{-2}) EGM2008+ECCO-2
EGM2008	360	62636854.25±0.06	62636854.29±0.02

5. DATUM CONNECTION

To estimate the offsets between the regional datums and the global datum, W_0 values obtained at TGs were averaged for each region or datum separately. The resulting W_0 values represent the regional or local LVDs within geopotential space. In fact, the more TGs are involved for each region, the better representation will be for the datum in terms of W_0. Eq. (4) was used to estimate offsets between the global and regional datums. Offsets shown in Table 3 in a unit of m^2s^{-2} are given an approximate metric presentation by means of the normal gravity acceleration at the surface of the reference ellipsoid (γ_0)

$$\delta H = \frac{\delta W_0}{\gamma_0} \qquad (5)$$

and presented in Fig. 4.These results show that Germany has the largest offset from the global datum of -1.3 m^2s^{-2} while Stornoway has the smallest offset of -0.45 m^2s^{-2}. These offset can be used to connect vertical datums in the involved regions into the world height datum.

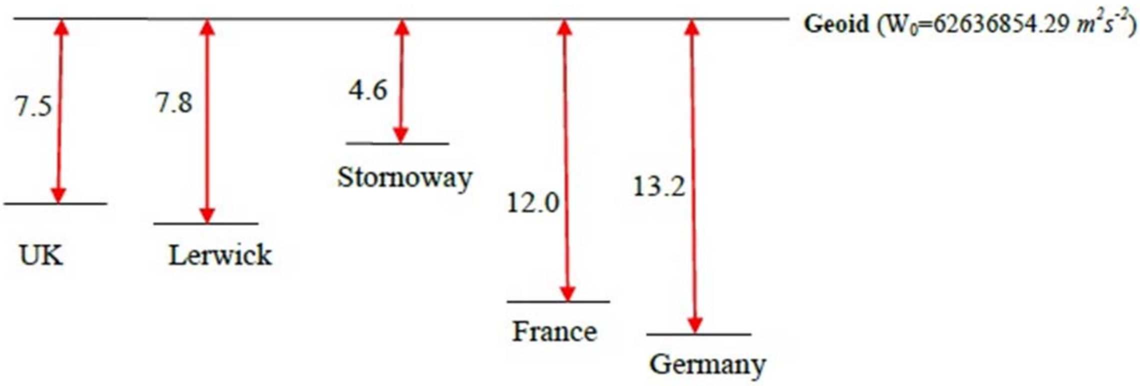

Figure 4: Datums offsets from the global datum in a unit of *cm*.

Table 3: Datum offsets.

Datum	δW_0 (m^2s^{-2})	δH (cm)
UK(mainland)	-0.74	-7.5
Lerwick	-0.77	-7.8
Stornoway	-0.45	-4.6
France	-1.18	-12.0
Germany	-1.30	-13.2

6. CONCLUSIONS

In this study, the possibility of unification of vertical datums within geopotential space has been investigated. In the regional study, TG, GPS, MDT and leveling information in the UK, France and Germany was used to construct geoid coordinates at the TG locations. The geopotential value at each location was estimated by means of EGM2008 to its maximum spectral resolution before and after the use of MDT. This shows that MDT plays an essential role in consistencies between the geopotential values and consequently between the regional datums.

This study also estimated W_0 globally using an altimetric MSL (CLS01), oceanographic MDT (ECCO-2) and an Earth gravitational model (EGM2008). Based on this part, an estimate of W_0=62636854.29 ± 0.5 m^2s^{-2} was assigned to the global vertical datum. Offsets of the regional datums from the global datum were computed. This revealed that the German and Stornoway datums have the largest and the smallest offsets from the global datum respectively.

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BIOGRAPHICAL NOTES

Dr. Nadim Dayoub is a teaching member at the Tishreen University in Syria. He has a PhD degree in Geomatics from the University of Newcastle in the UK and a Bachelor degree in Civil Engineering from the Tishreen University in Syria. During his recently started career he has worked as a lecturer in Topographic Engineering at the Tishreen University teaching Physical Geodesy and basics of GPS. His research interests are linked to Physical Geodesy; in particular he focuses on unification of vertical datums within geopotential space and investigating the geoid geopotential value and its time variations. His work was presented at a few international scientific meetings such as AGU2006 and AGU2009 in San Francisco, IAG2009 in Buenos Aires and EGU2010 in Vienna. His current research involves validating the Syrian GPS/leveling network through comparison with different geoid solutions. He is a member of The Association of Syrian Engineers and the Association of Syrian Teachers.

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