Fast and Convenient Determination of Geoid Undulation N in an Urban Area

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Key words: geoid, undulation N, urban area, fast method, permanent GPS station, accurate forward-backward trigonometric heighting

SUMMARY

The accurate determination of the geoid surface constitutes as one of the primary targets in the discipline of geodesy. The knowledge of this surface, namely the geoid undulation N or the altitude of the geoid, offers geodesists the opportunity to convert the geometric heights (h) of the earth's surface points, which are determined by using GPS (Global Positioning System) measurements and which have no physical meaning, into orthometric heights (H). Subsequently, in recent years, various methods have been developed. These methods are either based on the determination of components ξ and η of the deviation of the vertical at a point on the Earth's physical surface or on the determination of Δg changes of gravity. Even though, the methods mentioned above lead to the adequate calculation of the geoid undulation N within the order of few centimetres, they are very difficult to use and time-consuming, which makes them quite disadvantageous.

A direct method used for the accurate determination of the geoid surface is based on the knowledge of orthometric and geometric heights of uniformed distributed points in the area. This area should be of a limited scale (few km^2) and be of a relatively smooth ground. Thus, by using this method, the determination of the geoid undulation N mainly depends on the accurate determination of the orthometric and geometric (ellipsoidal) heights of the points, which were selected to be used. However, it also depends on the number of the available points, their distribution in the area and last but not least on the smoothness of the geoid surface at the region. Moreover, the values of the geoid undulation N can be utilised either for the compilation of a local geoid map or for the fitting of a suitable surface to them.

This paper presents one quick and primarily flexible and convenient method in order to determine the geoid undulation N in an urban area of a few km^2. Therefore, taking advantage of one existing leveling urban network, measurements were carried out on a selected and adequate number of benchmarks in order to determine their orthometric and geometric heights. The spirit leveling method and the equivalent method of the accurate forward-backward trigonometric heighting were used for the determination of the orthometric heights while the method of the relative static positioning was used for the geometric ones in reference to a permanent GPS station. The accuracy of the orthometric and geometric heights was of the order of ±1mm and ±2cm, respectively and lead to the determination of the geoid surface of this densely-built urban area within an accuracy of the order of a few centimeters. Three types of surfaces: the plane, the bi-linear surface and the 2^{nd} degree surface, were examined, to determine which is better fitting to the region by comparing their respective accuracies. The enrichment of the global geoid model EGM08 (Earth Geopotential Model 2008) was also attempted, in the determined area, with amazing results. The analysis of the results allowed us to come up with a series of invaluable conclusions regarding the success of this method and the accuracy of its results.

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1. INTRODUCTION

The GPS system altimetry is considered today to be a good alternative method instead of the classic terrestrial ones, since it is more economical, less time-consuming and moreover less tiring. Nevertheless, the basic disadvantage in using the GPS is the fact that the heights that are determined are geometric heights (h) that use the ellipsoid GRS ′80 of the geocentric World Geodetic System 1984 (WGS′ 84) as a reference surface. These geometric heights are of no natural significance, and so it is necessary to convert them into orthometric heights (H), which use the geoid as their reference surface instead. The heights are converted by using the well-known equation H=h–N, which employs the individual undulation value N of the geoid at each point. Therefore, this undulation value N must be known beforehand in order to utilize the above-mentioned relationship.

The undulation of the geoid N can be determined through astrogeodetic leveling, astrogravimetric leveling and by satellite altimetry. Furthermore, it can be determined by using the orthometric and geometric heights of a number of points as well as by using a combination of all the methods above. The next step after finding the undulation values N of the geoid, and consequently, the form of the geoid, is to create maps or geoid models on both a global and a regional scale.

The only available geoid model in many regions of the Earth is a global geoid model and not a regional one. However, the basic disadvantage of the present global geoid models is the fact that, as far as topographic work is concerned, there is a great uncertainty in the undulation values N that these models provide. More specifically, the EGM96 (Earth Geopotential Model 1996) global geoid model provided undulation values N with an uncertainty of ±37cm, while the values provided by the EGM08 (Earth Geopotential Model 2008) model had an uncertainty of ±22cm (Gomaa D., et al, 2010).

This great uncertainty of the global models has made it imperative for engineers to create regional geoid models in areas, which will provide undulation values with less uncertainty, ones that will not be more than a few centimeters. These regional geoid models can finally be used for the enrichment of the global ones, in order to make them more accurate in specific regions. Following that, it will be possible to convert the geometric heights at regional points to orthometric heights that will be accurate enough in order to satisfy each and every individual demand.

2. PROCEDURE

2.1. Determination of the orthometric heights

The determination accuracy of orthometric heights (H) is mostly dependent on the measurement method that has been used. At present, the accuracy of their determination may reach a few millimeters by mainly utilizing digital instruments as well as digital accessories.
The most frequently used method is the digital double spirit leveling method, which provides an accuracy of ±1-2mm. This method is considered to be the most accurate for determining orthometric heights, but at the same time, it is also considered to be the most time-consuming method. However, several factors have to be taken into consideration if this method is to be used in an urban and densely built area. It is possible that the actual level stations will be more than those theoretically calculated, since these areas always include unforeseen obstacles, and as a result, the theoretical leveling time can be significantly less than the actual one. Moreover, the passing of automobiles and pedestrian presence are of the major problems that have to be dealt with, since they impede on the entire procedure and increases the time of measurements. Finally, another serious problem faced in these urban areas, is the atmosphere itself, which affects the propagation of the signal, thus reducing the accuracy of the measurement.
Another method that provides the same-order accuracy with the digital spirit leveling is the method of the accurate forward-backward trigonometric heighting by using reflectorless total stations (Lambrou E., et al, 2007, Lambrou E., 2007). If the instruments are properly selected (Total station) and both the right number of crewmembers and the right positions for placing the instruments are selected, then this method constitutes as a very fast but also equally expensive method for determining orthometric heights in urban areas.

2.2. Determination of the geometric heights

Today, there are a few thousand GPS/GLONASS permanent stations of continuous operation that operate in more than 50 networks, in order to facilitate positioning and navigation applications at a global, continental, national or regional scale. The use a permanent reference station can reduce the amount of fieldwork required, the amount of equipment necessary to be present as well as the number of crewmembers needed on site, thus making field work both more time-efficient and more accurate.
The most accurate method for determining geometric heights of network points is the relative static positioning method, which can bring results with an uncertainty of a few centimeters. Measurements using this method are measurements of phase ambiguity of the carrier wave. The use of a two-frequency (L1 and L2) GPS receiver is an extra advantage when using this method. In this way, the ionospheric reaction, which is dependent on the frequency, can be dealt with more sufficiently.
Today, in order to make the above-mentioned method even more time-efficient and more flexible, there is the option of using a station from a permanent stations network. In this case, the only thing needed is a GPS system receiver (rover receiver), which will be successively placed at the points whose geometric heights are being researched. In this manner, there is less equipment required (only one receiver instead of two), a fact which is extremely important in projects administered in urban areas, since less equipment will also reduce the

time needed for the transportation of the equipment, thus making the entire procedure even more appealing. In this way, the time needed for the measurements as well as the cost entailed, is significantly reduced.

In order for this method to be kept quick, it is important to take two more factors into consideration. The first important factor is how the antenna is placed. The GPS receiver's antenna should be placed on a pole and not on a tripod, in order to minimize the time required. This pole must be vertically aligned with the help of both an integrated spherical level as well as another auxiliary safety level and stabilized by a light GPS antenna tripod. The second, but equally important factor to be taken into consideration is the moving receiver's duration of stay at each point. It must not exceed 20 minutes. In these types of urban and densely-built areas, the accuracy of determination cannot be improved even if the receiver stays at that point for additional hours.

Thus, if all of the above-mentioned requirements are followed for bases with length of about 4 Km, then the determination of geometrical heights will be realized with an accuracy of ±2cm, by using a much quicker method. Usually in such areas, some benchmarks are established and a leveling urban network is built.

The geometric heights (h) of the benchmarks must be determined by using GPS system measurements. However, a basic problem has to be overcome in order to carry out these measurements. Particularly when the placement of the receiver's antenna cannot be made directly to the benchmarks, as they are installed in the buildings' walls. The establishment of new temporary station points was chosen as the optimal solution at a distance of the order of 15m from each benchmark. As the distance between the benchmark and the temporary point was limited, it could be assumed that the difference of the geoid undulation was equal to zero ($\Delta N=0$), concluding that the orthometric height difference was equal to the geometric height difference ($\Delta H=\Delta h$). Thus, the GPS measurements were carried out on these new points and measured the orthometric height difference between the benchmark and the new point. The geometric heights of the benchmarks of the original network were determined by using the equation:

$$h_{benchmark} = h_{new\_point} - \Delta H_{benchmark\ -\ new\_point} \qquad (1)$$

Particular attention must be given to some main elements to achieve the above method. The finding of the locations that the new station points will be placed at, is a very important step, as in urban and densely-built areas the best possible visibility of the satellites must be ensured.

3. MAP PRESENTATION

The knowledge of both orthometric and geometric heights of a number of points can lead to the modelisation of a regional geoid. By having this regional model of the area's geoid, it is possible to determine the undulation value N of the geoid in other points in that same area. In this manner, by using only the GPS system measurements and determining the geometric heights of points, we are now capable of finding the corresponding orthometric ones.

The uncertainty by which these orthometric heights can be determined depends on the uncertainty of finding the corresponding undulation value N of the geoid from the model, but also by the accuracy of determination of the geometric heights measured by the GPS. The determination of the accuracy of N through a local geoid model is, in turn, related to the

accuracy of determination of the geometric heights of the points used for modelisation, as well as the interpolation method that will be used.
The most frequented way of presenting the undulation values of the geoid is by using a map of undulation contours N. In order to create such a map, it is necessary to have a satisfactory number of points that are equally distributed in the area, and that the undulation value N of these points is known. Finally, in order to produce such a map, it is required to choose the proper interpolation method (the most usual interpolation method employed is the Kriging method), as well as the interval, which is selected by taking into consideration the variation of the geoid in the region.

4. THE SURFACE FITTING

In the case of a region, which does not exceed a few Km^2, it is possible to find a certain surface that will be better fitting to the undulation values N of the geoid. The most frequently used surfaces are: the plane, the bi-linear surface and the 2^{nd} degree. It is necessary to know the undulation value N of the geoid of at least three points in that region, while knowing the undulation value N of the geoid for more points can result in determining the surface by using the least squares method.
When the area of interest is small and the geoid has a normal variation, then it is possible to approach it by using a plane with a mean slope. The equation for such a plane is given by the equation:

$$h_i - H_i = N_{f_i}(\varphi_i,\lambda_i)=a_0+a_1\cdot(\varphi_i-\varphi_o)+a_2\cdot(\lambda_i-\lambda_o) \quad (2)$$

where φ_i, λ_i :are the geodetic coordinates, latitude and longitude, of point i in the World Geodetic System 1984 (WGS′ 84),

φ_o,λ_o : are the geodetic coordinates, latitude and longitude, of a central point in the region of interest in the World Geodetic System 1984 (WGS′ 84),

N_{f_i} : is the undulation of the geoid in the point i, with regard to the reference ellipsoid

H_i, h_i: are the orthometric and geometric height in point i, and,

a_0, a_1, a_2: are the unknown parameters of the plane.

If more than three points are known, then the final results can be tested with regard to their reliability and accuracy, by using the standard error σ_o.
The above equation can be used in a manner corresponding to the one used for creating the map. Those that are interested can use this relationship in order to determine the undulation values N_{f_i} of the geoid in other points within the region of interest.
In the case where the region of study shows a more prominent surface, then there is the option of attempting to fit a bi-linear surface or a 2^{nd} degree area.
Respectively, in the bi-linear's case the corresponding equation is:

$$h_i - H_i = N_{f_i}(\varphi_i,\lambda_i)=a_0+a_1\cdot(\varphi_i-\varphi_o)+a_2\cdot(\lambda_i-\lambda_o)+a_3\cdot(\varphi_i-\varphi_o)\cdot(\lambda_i-\lambda_o) \quad (3)$$

But it is necessary to know the undulation value N of the geoid in at least four points in the region.
In the 2^{nd} degree surface the equation is transformed into:

$$h_i - H_i = N_{f_i}(\varphi_i,\lambda_i) = a_0 + a_1\cdot(\varphi_i-\varphi_o) + a_2\cdot(\lambda_i-\lambda_o) + a_3\cdot(\varphi_i-\varphi_o)^2 + a_4\cdot(\lambda_i-\lambda_o)^2 + a_5\cdot(\varphi_i-\varphi_o)\cdot(\lambda_i-\lambda_o) \quad (4)$$

In which it is necessary to know the undulation value N of the geoid at least in six points in the region.

4.1. Evaluation of the surface fitting

After finding a surface that fits to the values N of the geoid in the points of that region, the next necessary step is to evaluate the fitting of that surface. This evaluation is of immense importance, since it shows whether the calculated area is actually fitting to the region. For this purpose, it is necessary to test the validity of certain criteria.

Initially, the unknown parameters must be tested in order to assess whether they are statistically important for a certain confidence level (for example 95%). Each one of the unknown parameters a_i must conform to the following formula:

$$\sigma_{a_i} \cdot z_{95\%} \leq a_i \quad (5)$$

Where σ_{a_i} : is the standard deviation of each unknown parameter a_i and,

$z_{95\%}$: is the coefficient of the normal distribution for one dimension arrays for confidence level 95%.

If the equation (5) is valid, then the parameters a_i will be considered as being statistically important.

Another way to evaluate the fitting surface is to test the a-posteriori standard error of the weight unit $\hat{\sigma}_0$. By comparing these errors, the surface that has the smallest error of the weight unit $\hat{\sigma}_0$ is better fitting to the region of interest. Also the smaller the uncertainty for each determined parameter the better the fitting of the surface.

Finally, it should be investigated whether it is useful to find a surface of a greater degree. This test can be realized through the following formula:

$$\frac{r_1 \cdot \sigma_2}{r_2 \cdot \sigma_1} \leq F_{1,r_2} \quad (6)$$

Where r_1, r_2 : are the degrees of freedom of the smaller degree surface and the greater degree surface respectively,

σ_1, σ_2: are the standard deviations of the two surfaces fitting respectively, and,

F_{1,r_2} : The coefficient of the F distribution for one degree difference between the tested fitting surfaces.

If the equation (6) is valid, then it isn't practical to find a greater degree area.

5. THE METHOD'S ACCURACY

This method is based on the direct determination of the orthometric (H) and geometric (h) heights of a number of uniformly distributed points in an urban area of study, for the purpose of calculating the undulation values N of the geoid of these points. These points are then used in order to produce a geoid map or to find the equation of a certain surface that will be better fitting to these values.

The direct determination of the geoid undulation values N_{D_i} of these points is realized through the known relationship $N_{D_i} = h_i - H_i$. Thus, the uncertainty $\sigma_{N_{D_i}}$ of this relationship is dependent on the determination of the uncertainty of both orthometric and geometric heights, and is provided according to the variance – covariance law, by the equation:

$$\sigma_{N_{D_i}} = \pm\sqrt{\sigma_{h_i}^2 + \sigma_{H_i}^2} \quad (7)$$

Today, the uncertainty of determination of orthometric heights, through the use of proper instruments and methods, can reach ±1-2 mm; while correspondingly, the uncertainty of determination of geometric heights of urban regional points can reach a few cm. Therefore, the uncertainty of direct determination of the N_{D_i} will depend mostly on the uncertainty of the geometric heights that have been found.

The uncertainty, of the N_{f_i} values for other unknown points calculated by the determined fitting surface equation, $\sigma_{N_{f_i}}$ can be estimated by applying the variance – covariance law in the corresponding equation of the chosen surface, equations (2), (3) or (4).

Specifically, in the plane's case the uncertainty $\sigma_{N_{f_i}}$ is given by the aquation :

$$\sigma_{N_{f_i}} = \pm\sqrt{(\frac{\partial N_f}{\partial a_o})^2 \cdot \sigma a_o^{\;2} + (\frac{\partial N_f}{\partial a_1})^2 \cdot \sigma a_1^{\;2} + (\frac{\partial N_f}{\partial a_2})^2 \cdot \sigma a_2^{\;2} + (\frac{\partial N_f}{\partial \varphi_i})^2 \cdot \sigma \varphi_i^{\;2} + (\frac{\partial N_f}{\partial \lambda_i})^2 \cdot \sigma \lambda_i^{\;2} + (\frac{\partial N_f}{\partial \varphi_o})^2 \cdot \sigma \varphi_o^{\;2} + (\frac{\partial N_f}{\partial \lambda_o})^2 \cdot \sigma \lambda_o^{\;2}} \quad (8)$$

Considering that $\sigma_{\varphi_i} = \sigma_{\varphi o} = \sigma_{\lambda_i} = \sigma_{\lambda o} = \sigma_{\varphi}$, the previous equation is finally changed into:

$$\sigma_{N_{f_i}} = \pm\sqrt{\sigma a_o^{\;2} + (\varphi_i - \varphi_o)^2 \cdot \sigma a_1^{\;2} + (\lambda_i - \lambda_o)^2 \cdot \sigma a_2^{\;2} + 2 \cdot (a_1^{\;2} + a_2^{\;2}) \cdot \sigma\varphi^2} \quad (9)$$

6. APPLICATION

An experimental application was carried out at a test area in the center of Athens. The network consisted of 16 benchmarks, which covered an area of about 2km · 4km.

The orthometric heights (H) of the network points varied from 71m to 124m to the mean sea level and were determined by the spirit leveling or by the accurate forward-backward trigonometric heighting method (Lambrou E., 2007, Lambrou E., et al, 2007). The accuracy obtained by both of these methods was of the same order. Specifically, the benchmarks' height accuracies fluctuated from ±1mm to ±4mm and the rms by the leveling network was calculated at $\hat{\sigma}_0 = \pm 1\text{mm}$.

The result of the 16 network bases was attained by using one permanent reference station of the HEPOS (HEllenic POsitioning System) (www.hepos.gr) network as the fixed point of known coordinates. This point, named 98A, was situated at a distance of 4km from the tested area. The HEPOS used the geocentric ellipsoid GRS'80, according to the ETRS'89, in the frame ETRF05 epoch 2007.5. Therefore, the geometric heights (h_{new_point}) of the temporary

points were determined. The geometric heights ($h_{benchmark}$) of the original benchmarks were calculated by using equation (1). Specifically, the geometric height of the benchmarks varied from 110m to 162m and their uncertainties from ±6mm to ±23mm.
Finally, as the orthometric (H) and the geometric (h) heights of the points of the leveling network were known, the values of the geoid undulation N_{D_i} of the16 benchmarks could now be directly estimated. The fundamental formula $N_{D_i} = h_i - H_i$ was used to determine the values of the geoid undulation and equation (7) was used to calculate the accuracy which ranged from ±10mm to ±23mm. Figure 1 illustrates the local geoid map. The relative geoid undulation between the network points range up to 20cm.
It was decided to approximate the geoid surface by using either a plane equation, a bi-linear equation or a 2nd degree equation.
Various tests were performed to find the equation that best suited the surface of the examined area. Since the known points were more than the required (only 3 known points were required), more observation equations were formed. Thus, the result was reached by using the least squares method. However, we did not use all 16-network points to determine the values of the geoid undulation N in order to use the points that were not selected in the determination of the equation for control purposes.

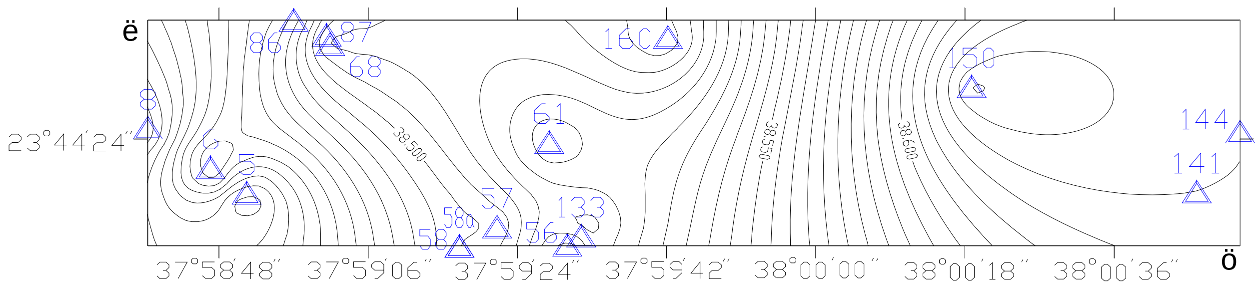

Figure 1. Local geoid map / Interpolation method: Kriging/Contours interval: 5mm

Several points of the leveling network were selected to cover the whole area. Hence, the parameters of the plane equation, their corresponding uncertainties and the standard deviations of the adjustment are identified by using 9, 11, 12 and 13 points. They are all presented in the table 1. Particular attention was paid to the spread of the points so that the area of study was covered in a uniformed way.
For each of the above planes, a check of the significance of the unknown parameters a_0, a_1 and a_2 was made using equation (5). For all the four planes, which were formed, it was found that indeed the coefficients were statistically significant.

	***9* points**	***11* points**	***12* points**	***13* points**
$\mathbf{a_0}$	38.5202 ± 0.0035	38.5227±0.0033	38.5201±0.0042	38.5259±0.0071
$\mathbf{a_1}$	243.9779 ± 20.8278	252.0681±17.5672	267.9822±22.4532	287.6797±38.9365
$\mathbf{a_2}$	175.7994±64.8970	140.6205±63.3866	179.3156±83.4035	339.5699±134.3863
σ_o (cm)	1.0	1.1	1.5	2.6

Table 1. The parameters a_0, a_1 and a_2 of the planes, their rms and the σ_o of each plane equation.

Considering the above, it was observed that the plane equation that was formed by using 9 points had the smallest σ_o. However, we finally selected the plane equation formed by 11 points as it offered a better spread. The standard deviation error was equally small and provided smaller uncertainties for each determined parameter. The following plane equation was reached after applying equation (2):

$$h_i - H_i = N_{f_i}\ (\varphi_i,\lambda_i) = 38.5227+252.0681\cdot(\varphi_i-\varphi_o)+140.6205\cdot(\lambda_i-\lambda_o) \qquad (10)$$

The standard deviation of the chosen solution was $\acute{o}_0 = \pm 11\text{mm}$. Applying the variance – covariance law according to equation (9), the geoid undulation N was calculated for other random points in this area by an average uncertainty of $\sigma_{N_f} = \pm 5.6\text{mm}$.

It was also decided that a map be created that would present the uncertainty σ_{N_f} of each random point in the test area. Figure 2 illustates the map of the uncertainty σ_{N_f}.

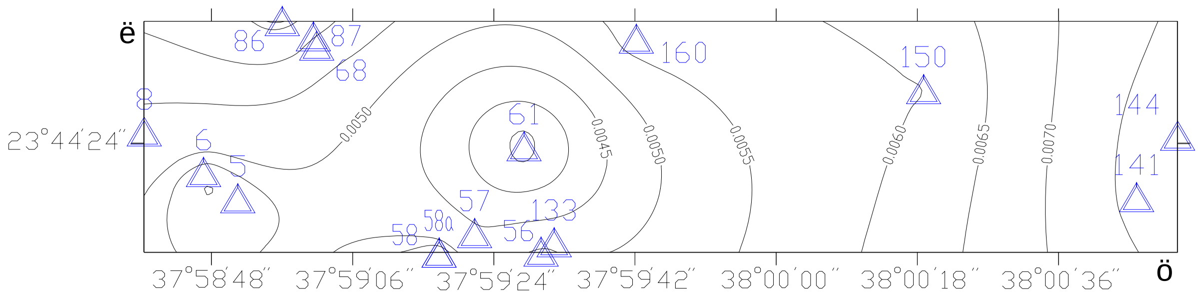

Figure 2. Local map of the uncertainty σ_{N_f} / Interpolation method: Kriging/Contours interval: 0.5mm

New values of the geoid undulation N_{f_i} were calculated according to equation (10). For the remaining 5 known points of the leveling network and their orthometric heights the basic equation $H_{f_i} = h_i - N_{f_i}$ was used. This calculation served as a quality control of each approximation and is presented in table 2 (control points are in bold letters).

The uncertainty of the δH is $\sigma_{\delta H} = \pm\sqrt{\sigma_H^2 + \sigma_{H_{f_i}}^2}$ where $\sigma_H = \pm 0.2\text{cm}$ and $\sigma_{H_{f_i}} = \pm\sqrt{\sigma_h^2 + \sigma_{N_f}^2}$ ($\acute{o}_h = \pm 2\text{cm}$ and $\sigma_{N_f} \approx \pm 0.6cm$), that means $\sigma_{\delta H} = \pm 4.1\text{cm}$ for confidence level 95%.

All the differences in the calculation of the orthometric heights of the points are within this fluctuation.

An attempt was also made to adapt a bi-linear and a 2nd degree surface. It was observed that both of the adaptations were not statistically correct probably due to the small size of the tested area.

Point	Orthometric height H (m)	Geometric height h (m)	N_{D_i} direct geoid undulation N values (m)	$N_{f i}$ by the plane model (m)	$H_{f_i} = h_i - N_{f_i}$ by the plane model (m)	$\delta H = H - H_{f_i}$ (cm)
	(1)	(2)	(3)	(4)	(5)	(6)=(1)-(4)
5	**85.760**	**124.198**	**38.438**	**38.474**	**85.724**	**3.6**
6	88.256	126.731	38.475	38.471	88.260	-0.4
8	**91.351**	**129.785**	**38.434**	**38.466**	**91.319**	**3.2**
56	75.934	114.460	38.526	38.517	75.943	-0.9
57	75.780	114.283	38.503	38.508	75.775	0.5
58	70.747	109.245	38.498	38.501	70.744	0.3
58A	**70.746**	**109.246**	**38.500**	**38.501**	**70.745**	**0.1**
61	83.163	121.689	38.526	38.524	83.165	-0.2
68	104.513	143.021	38.508	38.500	104.521	-0.8
86	123.488	161.971	38.483	38.497	123.474	1.4
87	113.189	151.696	38.507	38.501	113.195	-0.6
133	78.694	117.205	38.511	38.520	78.685	0.9
141	89.498	128.121	38.623	38.615	89.506	-0.8
144	96.825	135.449	38.624	38.626	96.823	0.2
150	**105.176**	**143.802**	**38.626**	**38.591**	**105.211**	**-3.5**
160	**99.882**	**138.385**	**38.503**	**38.551**	**99.834**	**4.8**

Table 2. Orthometric and geometric heights of the leveling network points by using direct and local model calculations.

New values of the geoid undulation N were determined by the global geoid model EGM08. These values were calculated by using the program "harm_synth" (created by Pavlis N.), in TideFree system, in which only the moon's and the sun's tides have been removed.

Finally, it was decided to adjust the values of the undulation N which had been determined by the global geoid model EGM08 with those values which were directly determined by using orthometric and geometric heights. This procedure is used for the global model to become enriched with terrestrial data. Hence, the global model can provide more correct values of the geoid undulation N, as the systematic errors and the local abnormalities are calculated and extracted. The following model was used for this adjustment:

$$\delta N_i = N_{D_i} - N_i^{EGM08} = (h_i - H_i) - N_i^{EGM08} = a_0 + a_1 \cdot (\varphi_i - \varphi_o) + a_2 \cdot (\lambda_i - \lambda_o) \quad (10)$$

Where

N_{D_i} : is the direct geoid undulation in the point i, with regard to the reference ellipsoid

N_i^{EGM08} : is the geoid undulation in the point i, with regard to the reference ellipsoid, by the global geoid model EGM08.

The end result was reached by using the least squares method and led to the following equation:

$$\delta N_i = N_{D_i} - N_i^{EGM08} = -0.5898+190.6845\cdot(\varphi_i-\varphi_o)-113.5694\cdot(\lambda_i-\lambda_o) \quad (11)$$

The standard deviation of the adjustment was $\hat{\sigma} = \pm 2.2cm$ and the rms of the three unknown parameters were $\sigma_{a_0} = \pm 0.005m$, $\sigma_{a_1} = \pm 26.8971m$, $\sigma_{a_2} = \pm 97.3550m$.

The above method essentially calculates a correction surface, which reflects the adjustment which must be applied to the EGM08 geoid undulation N to coincide with the corresponding geoid undulation N which was determined directly.

Point	N^{EGM08} direct by the EGM08 (m)	N_{D_i} direct geoid undulation N values by terrestrial data (m)	δNi by the equation (11) (m)	$N_{enrichment}^{EGM08}$ after the enrichment with terrestrial data (m)	$\Delta N^{EGM08} = N_i^{EGM08} - N_{57}^{EGM08}$ (mm)	$\Delta N_D = N_{D_i} - N_{D_{57}}$ (mm)	$\Delta N_{enrichment}^{EGM08} = N_{i,enrichment}^{EGM08} - N_{57,enrichment}^{EGM08}$ (mm)	$\delta N_{new} = N_{D_i} - N_{enrichment}^{EGM08}$ (mm)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)=(2)-(4)
5	39.084	38.438	-0.620	38.464	-9	-65	-39	-26
6	39.086	38.475	-0.626	38.460	-7	-28	-43	15
8	39.088	38.434	-0.636	38.452	-5	-69	-51	-18
56	39.094	38.526	-0.580	38.514	1	23	10	12
57	39.093	38.503	-0.590	38.503	0	0	0	0
58	39.088	38.498	-0.592	38.496	-5	-5	-8	2
58A	39.088	38.500	-0.592	38.496	-5	-3	-8	4
61	39.108	38.526	-0.590	38.518	15	23	15	8
68	39.111	38.508	-0.622	38.489	18	5	-14	19
86	39.113	38.483	-0.627	38.486	20	-20	-18	-3
87	39.112	38.507	-0.623	38.489	19	4	-14	18
133	39.096	38.511	-0.580	38.516	3	8	13	-5
141	39.140	38.623	-0.514	38.626	47	120	123	-3
144	39.151	38.624	-0.514	38.637	58	121	134	-13
150	39.140	38.626	-0.547	38.593	47	123	90	33
160	39.130	38.503	-0.584	38.546	37	0	42	-43

Table 3. The enrichment of the geoid model EGM08 by using terrestrial data.

It is obvious by table 3 that a systematic error of about 50cm resulted between the terrestrial measurements and the values of the geoid undulation N which came from the EGM08. This big difference is not only the result of the bad fitting of the EGM08 to the specified area but also the result of the different definition of the zero geoid surface that the global model and the local datum use for the determination of the orthometric heights' The ΔN values which are counted in the (5th), (6th) and (7th) columns of table 3 are no longer affected by any systematic error. According to these three columns, the global model EGM08 improved after the enrichment, as the difference between the new values $N_{enrichment}^{EGM08}$ and the directly calculated ones N_{D_i} were of the order of about 1.5cm while before the enrichment they had been of the order of 6 cm.

7. CONCLUDING REMARKS

- The geoid undulation N can be determined directly in an urban area by an uncertainty of about ±2cm.
- The accurate forward-backward trigonometric heighting provides an equivalent determination of the orthometric height differences with spirit leveling, in half the time intervals, at specified urban areas.
- The use of permanent network stations leads to a fast determination of the geometric height in an urban area by using a double frequency receiver mounted on a pole.
- Twenty minutes of observation is enough for the determination of h with an uncertainty of ±2cm in a densely-built area. Thus, one can create a reliable geoid map or a local geoid model within a 6 hour measurement period for a correspond area.
- The procedure of the hypsometric connection is very easy and accurate when using spirit leveling of one benchmark with a very close (up to few meters) GPS point station. Thus, now benchmarks can obtain geometric heights (h). As benchmark leveling networks have already been established in urban areas, it is now convenient to equip them with geometric heights (h).
- The determination of local geoid models is now indispensable especially in urban areas where more infrastructure projects are being carried out.
- The procedure of the fitting of a plane surface in a specified area proved to be adequate and can be used for the majority of infrastructure projects.
- To select the best fitting surface, certain criterion, like the rms of unknown parameters a_0, a_1 and a_2 must be defined apart from the $σ_0$ of the adjustment. The most preferable being where this rms is smaller which means a better determination of these parameters.
- The enrichment of the EGM08 by using the available terrestrial data really improves the global model's result. The differences between the EGM08 and the terrestrial data after the enrichment are of the order of about 1.5 cm. Before the enrichment the difference was about 6cm.
- The large differences that appeared between the EGM08 and the local geoid undulation N calculations are probably due to the systematic difference, of about 50 cm of the geoid zero surface that they use.
- It is useful for every country to be able to refer to an entire hypsometric datum. Also, it is appropriate to determine the difference between the zero reference surface which every country uses for the determination of the orthometric heights in relation to the zero geoid surface of the global model EGM08.This consists systematic error between the geoid undulation N calculation by the model and the terrestrial measurements. If the differences ΔN are calculated, then they are free of this error.

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