A New Algorithm to Measure The Details on Objects’ Surfaces Using Digital Close-Range Photogrammetry

Ahmed Ahmed ELSONBATY and Mostafa Abdel-Bary EBRAHIM, Egypt

Key words: Computer model, Close-Range Photogrammetry, Measurements, Algorithm, Spatial coordinates

SUMMARY

The creation of a computer model of an object depends mainly on determining the spatial coordinates of a set of points on its surface that describes every detail of the object. Once a 3D computer model is created, it can be imported into a 3D visualization software, e.g., AutoCAD, 3D Studio, to show the model from different views with its real shape and details laying on its surfaces. Assigning a camera within the 3D visualization software enables several photos to be taken for the computer model. Measurements can then be taken mainly for the points that have been used to create the model, but the details laying on the created computer model’s surfaces cannot be measured unless there is an additional condition. The main aim of this research is to derive a new method to measure the details laying on the object’s surfaces. This can be done by measuring the spatial coordinates of the points describing the interested details on the visible part of the object’s surfaces from an image that has been taken for the computer model by an assigned camera. Since the curved surfaces are extensively used in architecture (such as cylinders, cones, and hyperboloids), this research considered object surfaces that appear in the photo as quadratic. The information of the set of points that used to create the computer model along with the assigned camera parameters are used as a source data for measuring the details laying on the object’s surfaces. The results of the new algorithm are obtaining any information, e.g. spatial coordinates, distances, volumes, for those interested details that laying on the computer model's surfaces.

A New Algorithm to Measure The Details on Objects' Surfaces Using Digital Close-Range Photogrammetry

Ahmed Ahmed ELSONBATY and Mostafa Abdel-Bary EBRAHIM, Egypt

1. INTRODUCTION

Three-dimensional models of physical objects are rapidly becoming more affordable in many fields such as inspection, navigation, object identification, visualization, and animation (Abdelhafiz A., 2009). Photo realistic computer models obtained from digital photogrammetric techniques are beneficial for several applications. Photogrammetric techniques offer a large potential for the solution of a wide range of measurement tasks in different fields. For certain applications, photogrammetric techniques have meanwhile been accepted as standard measurement techniques (Maas H. and Hampel U., 2006). One of the most important application using these measuring techniques is the documentation of architecturally significant historical buildings and world heritage (Grimm A. et al, 2001).
The process of generating 3D models consists of several well-known steps: capturing, points recovering, surface reconstruction, texture mapping, and visualization. At the capturing and points recovering steps, the image-based approach, photo-based scanner, and range-based approach (3D laser scanner) can be employed.
The image-based technique consists of the following steps:

- Photographing at least two images.
- Determining the interior and exterior orientations of the captured images.
- Measuring the points of interesting features in the images and computing the spatial coordinates of the measured points.

Two years ago, a new technique called "photo-based scanning" was presented in the field of digital photogrammetry by Eos System Inc. This technique compares two photos on a patch by patch basis to find the best matches. When these optimal matches are found, the already-computed position and orientation information for the photographs is used to compute the location of that patch in 3D space. When a regular grid of patches is sampled in image 1 and matched to the optimal image positions in image 2, the result is a dense cloud of 3D points. For the limitations of this technique, please refer to Walford A., 2010.
The range-based technique is based mainly on using a laser ray to measure distances. In the laser scan technology, a kind of laser scanner sweeps a laser beam over the object and times how long it takes to return, which provides the distance from the scanner to every sampled point (Walford A., 2010).
To make measurements of a 3D computer model's surfaces, only the points used in creating the model can be measured. The rest of the surfaces' details cannot be measured as there is no available information about these points unless their image coordinates.
Several trials have been done to achieve this target, but each has its limitations; for example, Krause K, 1996, Forkert and Gaisecker, 2002, Jansa et al, 2004, and Abdelhafiz A., 2009.
In this research work, we developed a new algorithm to measure the details on the surfaces of computer model. The measurements will be taken from an image that has been taken for the

computer model by an assigned camera within 3D Studio software. The points that used in creating the computer model and the assigned camera parameters within 3D studio software will be used as source data for determining the spatial coordinates of the interested details.

2. RESEARCH OBJECTIVE

The objective of this research is to measure the spatial coordinates of the descriptive points of the interested details laying on computer model's surfaces. Computer models can be imported into a 3D modeling software, such as AutoCAD or 3D Studio, for further technical handling. Some of the techniques for creating 3D models can provide the real texture of the model, while others cannot. For those techniques that provide only shaded surfaces, the 'digital projector approach' can be applied to obtain the computer model with its real details on its surfaces (Hanke K., Ebrahim M., 1999).

Once the computer model is ready with its real shape, a camera could be assigned with known interior and exterior orientations and still images for the object could be taken from any location. The taken still images parameters (the interior and exterior orientations) will be saved in a database with the information of the points used in creating the model. Using the saved information, the spatial coordinates of any details laying on the model surface could be computed. Figure (1) presents the schematic chart of the system.

Ebrahim M. and Elsonbaty A., 2008 have derived a method to determine the spatial coordinates of such points, but depends on the assumption that the computer model is consists of planar faces. Because actual surfaces of objects are not restricted to planes, i.e., they can be cylindrical, conical, spherical, hyperbolical, etc., their solution is only approximate.

In an image, such as that shown in Figure (2), the object can consist of several surfaces with different geometrical types: spherical dome, cylindrical minaret with conical top, etc. Therefore, for such cases, the surfaces that can appear in a photo are of the second degree. Once the hidden parts are excluded from the scene, the surface upon which a point lies, as well as its equation and spatial coordinates, can be easily determined.

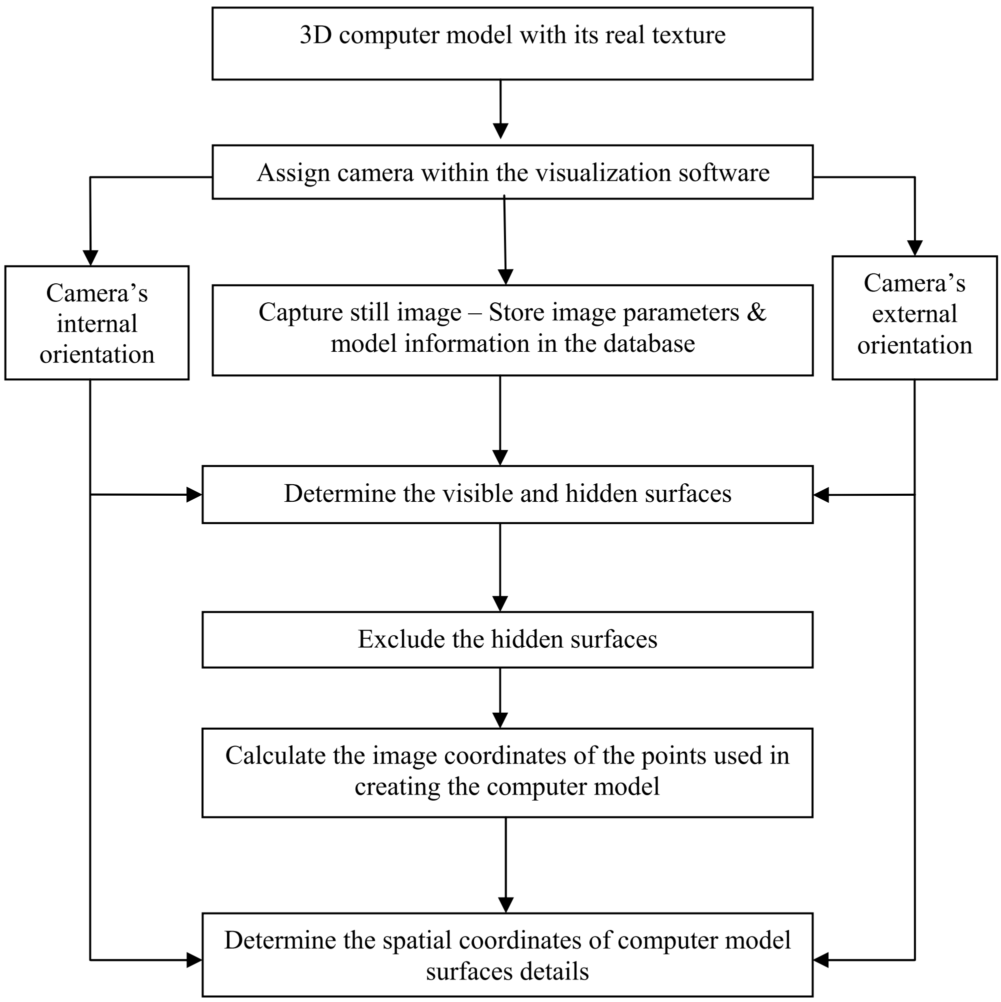

Figure (1) Schematic chart of the system.

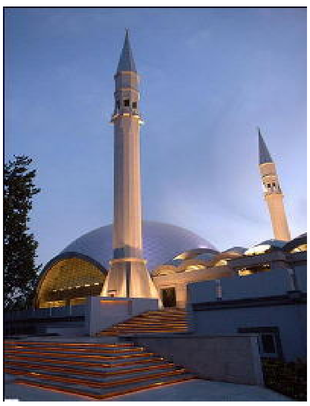

Figure (2) Modern architecture.

3. BRIEF DESCRIPTION OF THE METHOD

Assigning a camera within the 3D Studio software enables several photos to be taken for the computer model with known camera parameters, interior and exterior orientations. The 3D computer model's database, which contains all model information, points, edges, and surfaces along with the assigned camera parameters, will be used as source data for the new algorithm to measure any details appear in the image of the computer model that has been taken by the assigned camera. The ability to measure from a single image could be achieved when an additional condition is considered (Elsonbaty A., Ebrahim M., 2002). Such condition can be the surface where the interested dedails is laying on. Using the exterior and interior orientations of the assigned camera, the information of the hidden parts of the surfaces can be excluded from the computer model's database. The spatial coordinates of the object's details can be measured using the photogrammetric techniques after determining the surfaces where the interested details are laying on.

A brief description of the used method to solve the problem is given hereunder:

1. Determining the visible parts of the surfaces in the image plane:

   To determine the visible parts, there are four basic steps as follows:
   - Derive the transformation formulas to calculate image coordinates from spatial coordinates and vice versa using the photogrammetric techniques.
   - For one surface, find the points that laying on its visible side from the given database. Next, construct a closed polygon surrounding these points as a convex hull. This procedure will be done for each surface Φi. So, in the image plane, ***m*** polygons can be obtained depending on the number of surfaces appearing in the image.
   - Find the intersection points between these polygons, if any. Divid each polygon into parts through these points. Hence, divid the boundary of each surface in the image plane into a set of open polygons with the intersection points as start points and also end points in the list of points of each polygon.
   - Determine the visible parts of each surface, taking into consideration all surfaces together, by applying the theories of visibility. The visible parts of a surface may be represented by one or more pieces.

2. Determining the corresponding surface by determining the visible piece of area in which the point is located. The spatial coordinates of this point can now be calculated from the equation of that surface.

To carry out the above procedure, the spatial object has to be represented in a way that gives a complete and unambiguous definition of the object including not only the shape of the boundaries, but also the object's interior and exterior regions. In the following section, a brief explanation about the way in which objects will be represented on the computer will be given.

4. **REPRESENTATION OF OBJECT**

The object in space can be defined as set of surfaces Φi (i=1, 2,…, n). The points in each surface can be stored in one list of points. Each surface is enclosed by a closed polygon because only parts of each surface may be visible in the image plane. Hence, a system that is able to represent every element in the object must be defined. Such elements, which containing the needed sufficient information for solving the problem are points, polygons, and faces.

4.1. Data Structure for Points

The data structure of a point has point record number (*id*), its spatial coordinates (*P*), its image coordinates (P^c), and the address of the next point (*next*) as shown in Figure (3) (Elsonbaty A. and Stachel H., 1997).

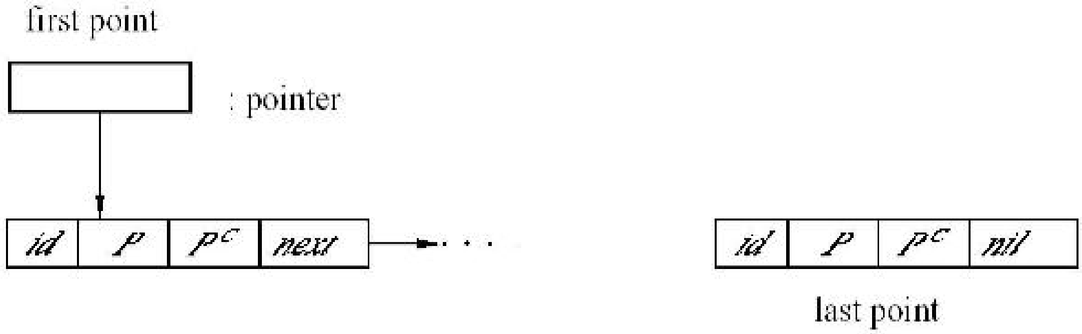

Figure (3) Data structure for a point.

4.2. Data Structure for Polygons

A polygon is defined by a sequence of the included vertices (*loop*). Therefore, the data structure of a polygon includes the polygon record number (*id*), record number of the surface on which it lay on (*id-S*), the pointer to its vertices (*Point-L*), and the pointer to the next polygon (*next*), as shown in Figure (4).

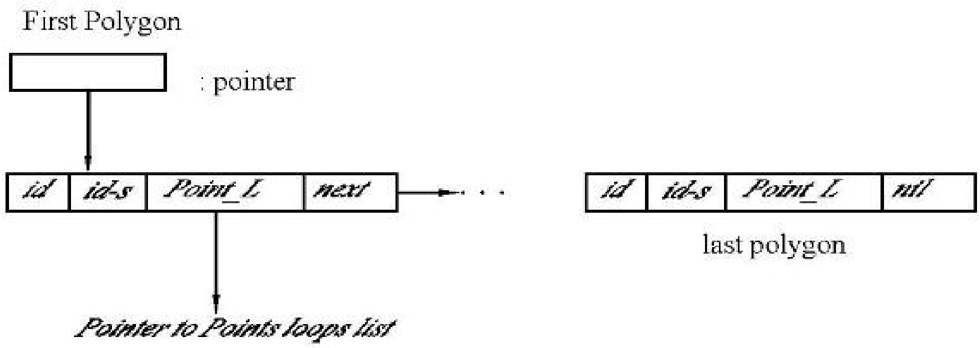

Figure (4) Data structure for a polygon.

4.3. Data Structure for Surfaces

The object surfaces are defined by a certain number of polygons. Each polygon is defined by a sequence of included vertices (*loop).* The outer loop of a certain face is processed mathematically as positive (anticlockwise), while the inner side is treated as negative (clockwise). For each surface, the following information needs to be stored as shown in Figure (5):

- The surface record number *(id),*
- A character indicating the type of the surface (cylinder, cone, etc.) (*Ty*),
- The pointer to the list of all points that located on it (*S_P_L*),
- The pointer to all points laying on its boundary in the image plane (*S_PL_O*),
- The pointer to the list of all visible polygons of the surface considered alone in the photo (*S_Poly1*),
- The visible pieces of the surface with other surfaces superimposed (*S_Poly2*), and
- The pointer to the next surface (*next*).

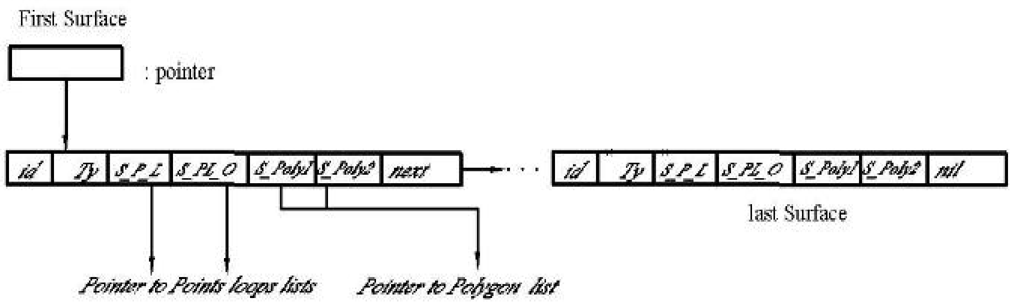

Figure (5) Data structure for a surface.

5. DETERMINING THE VISIBLE PARTS OF A SURFACE

To determine the visible parts of a surface, the following steps will be performed:

Step 1: From the 3D computer model's database, the points will be divided into ***n*** groups, each for one of the surfaces. This division will be done as follows:

The equation of one of the surfaces will be determined by selecting manually a set of points located on it. Then, all points in the database will be tested to determine the points that fulfilled the obtained equation of the surface within allowable errors.

As we are interested only in second-degree surfaces, the general equation can take the following form (Elsonbaty A., 2008 (a) & Farin G, 1990):

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{23}yz + 2a_{13}xz + 2a_{14}x + 2a_{24}y + 2a_{34}z + a_{44} = 0 \quad (1)$$

where a_{11}, a_{22}, a_{33}, a_{12}, a_{23}, a_{13}, a_{14}, a_{24}, a_{34} and a_{44} are constant coefficients, and at least one of the first six are non-vanishing. If we choose ***m*** points located on that surface and substitute their coordinates in equation (1), ***m*** equations in nine unknown coefficients can be obtained.

The ***m*** equations can be written in matrix form as:

$$\underset{(mxu)}{A} \; \underset{(ux1)}{X} = \underset{(mx1)}{B} \tag{2}$$

where *X* is the vector of the unknown coefficients, ***u*** is the number of unknowns (coefficients), ***m*** is the number of points (must be greater than nine), *A* is the matrix of the constant coefficients, and *B* is the vector of the constant term.
Equation (2) can be solved using the least squares technique to determine the coefficients in equation (1).
The points located in the database, which satisfy the surface equation, can now be extracted and stored in the *S_P_L* list of that surface.

Step 2: The polygon representing the boundary of the surface will be determined using the following procedure:

- At first, the visible points on a surface that appeare in the image will be determined by assuming that this surface is the only one appearing in the image plane. So, the polar plane of the surface with respect to the known center of projection will be determined (camera station) $S(x_0 : y_0 : z_0)$. The equation of the polar plane has the form:

$$a_{11}xx_o + a_{22}yy_o + a_{33}zz_o + a_{12}(xy_o + x_o y) + a_{23}(yz_o + y_o z) + a_{13}(xz_o + x_o z) + a_{14}(x + x_o) + a_{24}(y + y_o) + a_{34}(z + z_o) + a_{44} = 0 \tag{3}$$

  Because the camera station is outside the object, the points in *S_P_L,* which were located between the camera station and the polar plane, are visible. This determination could be achieved by substituting the coordinates of any point $P \in S\_P\_L$ into the equation (3). If the sign of the result is equal to that of the coordinates of camera station or zero, then the point is visible. These points will be stored in *S_PL_O*. Figure (6) shows the visible points located between the polar plane and camera station.

- The image coordinates for all points in *S_PL_O* will be calculated as shown in *Appendix I.*

- The points that located on the boundary of the surface in the image will be determined. These points formed the convex hull of the visible points that are on the surface in the image plane and stored in the *S_PL_O* list (Elsonbaty A, 2008(b)). The other points from

S_PL_O list will be then deleted. Figure (7) shows the points located on the boundary of the surface.

- Now, the rest of the points in this list formed a closed polygon surrounding the surface in the image.

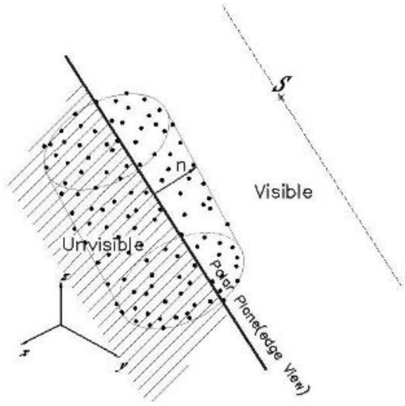

Figure (6) Visible points located between the polar plane and camera station.

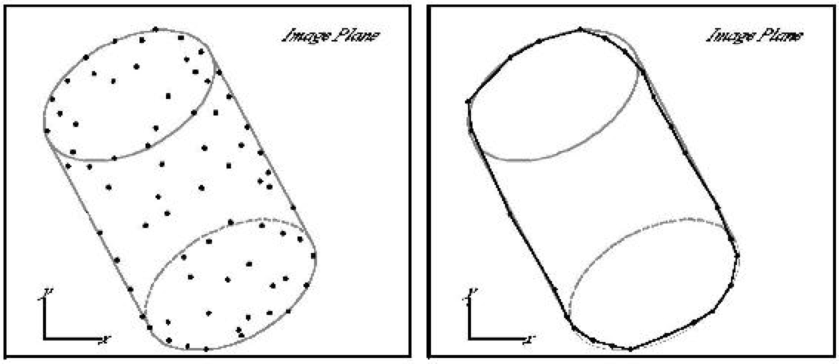

Figure (7) Points located on the boundary of the surface.

After repeating the above procedure for all the model's surfaces, we can obtaine several closed polygons in the image plane; two of them may look like those in Figure (8).

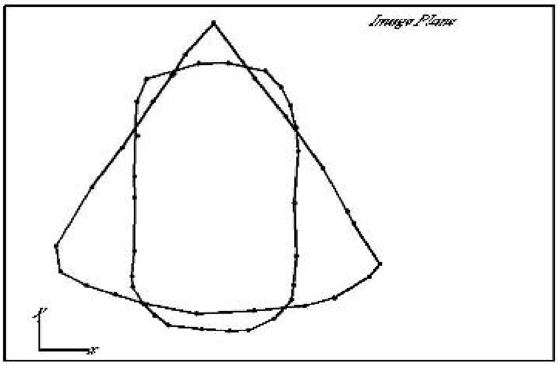

Figure (8) Boundaries of two surfaces in the initial stage.

Omitting of the hidden parts on each polygon: until this step, each surface is considered as appearing alone in the image plane. In this step, the images of all surfaces will be superimposed in one image.

Consequently, parts of the identified polygons in the previous step may be covered by another surface, as shown in Figure (8). To determine the hidden parts, the points of intersection T_i^c of two polygons must be found. To find these points, each polygon will be divided into a certain number of branches of open polygons, which are stored in *S_Poly1*. The corresponding points of T_i^c are the images of both two points that each one is located on one of the surfaces.

Starting with the point F^c which has the minimum y-coordinate in the image plane, the polygon to which it belonges will be determined. From this polygon, only the branch containing F^c and close to the nearest points of intersection will be considered. Three other branches started at any of the endpoints T_i^c. One of them belonged to the same surface, while the other two belonged to the other surface. Then, the outer polygon must be visible. The corresponding points T_i^c ($T_{i\phi 1}, T_{i\phi 2}$) on the two surfaces will be calculated in space. Figure (9) shows the visable parts on each polygon.

The surface that contained the nearest point to the camera station *S* is visible. The branch belonges to this surface is visible too. The other hidden ones will be then deleted from the list *S_Poly1*.

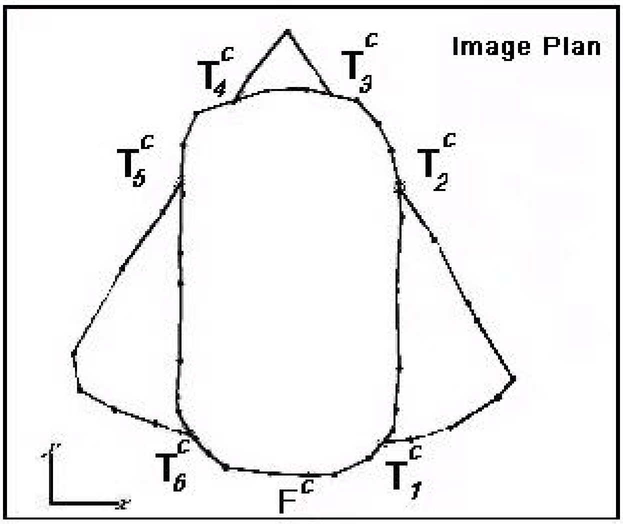

Figure (9) Hidden parts on each polygon.

Note: In some cases, the coordinates of $T_{1\phi 1} and T_{1\phi 2}$ are the same. This event happened when the point T_i^c is the image of a real point of intersection of the two surfaces, then the branch that is located inside the projection of their curve of intersection is visible, while the other is hidden.

When this procedure is done for each pair of polygons, there will be only three existing visible branches at each point of intersection T_i^c.

Determining the visible parts of each surface: each surface now has a list of branches stored in *S_poly1,* which representing the boundary of the visible parts of the surface. These branches are in general open polygons. The projection of the visible parts of the surface is determined as regions surrounded by closed polygons. This projection could be accomplished by doing the following steps:

1. The biging will be with any branch (*PL*) which located on the boundary of the object. This list is stored in *S_poly2* list.
2. Determine the surface (ϕ_1) to which this branch belonges.
3. Find all branches connected to surface (ϕ_1) at one of the end points T^c in the PL list. There are two lists of branches, PL_1 and. PL_2. Find their corresponding surfaces.
4. If one of the branches PL_1 or PL_2 belonges to the same surface (ϕ_1), then this branch will be added to the *S_Poly2*-list of each surface or both branches will be belonge to the other surface (ϕ_2). The branch located on the boundary of the object must be then excluded and the other branch will be added to the *S_Poly2*-list of each surface as shown in Figure (10).

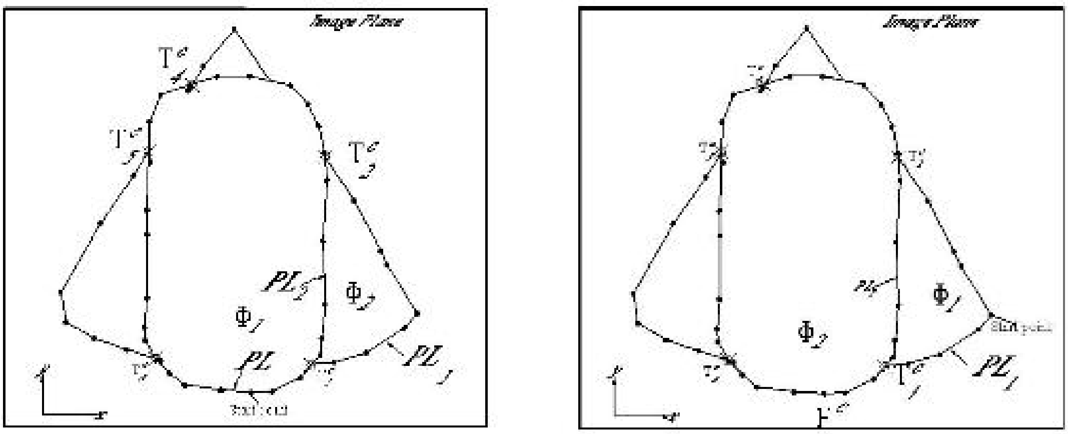

Figure (10) Determining visible parts of the surfaces.

Step 3: The visible areas of each surface (ϕ_i i) in the image are now represented by a set of closed regions surrounded by polygons stored in the list *S_Poly2*. To determine which surface containes the point *P*(in space), it is necessary to test whether its image P^c is laying inside one of the represented regions surrounded by the polygons in $PL \in S\_Poly2$. This determination is described in detail in (Ebrahim M. and Elsonbaty A., 2008). Once the region to which P^c belonges is determined, the surface of *P* and its equation will be also known. Using the image coordinates of P^c and the equation of the surface, the coordinates of *P* can be then easily calculated.

6. EXAMPLES

Considering an object restitution, first a complete reconstruction of the object's shape has to be done using e.g. any digitally matching method (e.g. Streilein, 1995) or a bundle adjustment program (Kager, 1980) followed by a definition of lines and surfaces. A few of the already before used images are then chosen to do a "Digital Slide Projection" onto the surface to achieve a restitution of the object details on its surfaces (Ebrahim, 1998). The surfaces of the objects are not restricted to planes, but are also of regular curved (e.g. cylinder etc.) and even irregular or free formed shape (Hanke and Ebrahim, 1996).
The spatial coordinates of the points that have been used to create the model along with the assigned camera parameters within 3D Studio can be used as source data to measure the spatial coordinates of the other model surfaces details' using the new algorithm.
In the following subsections, some computer models of real objects will be presented as examples of the 3D model which can be used to test the new algorithm. Testing the new algorithm in details comparable to the other surface measuring methods will be done in future research. The expected accuracy will remain within the accuracy of the surrounding points that used to determine the visible parts of the surfaces.
In addition, some photos of the objects' models will be shown, e.g. the objects' wireframe, shaded model, and the computer model image taken by an assigned camera in 3D studio with object real shape after applying the digital projector approach (Ebrahim, 1998).

6.1 Otto Wagner's Train Station (Vienna, Austria)

Otto Wagner's Train Station buildings on the Karlsplatz in Vienna has been selected in 1990 as a small test object, photographed, measured, and documented, in order to get well-checked materials to train students and photogrammetrists as well as to valuate internationally the results of the analytic photogrammetric amount of control information. The test object was a masterpiece of Art Nouveau, built in 1898-1899 (Waldhäusl, P., 1991). Figures (11) & (12) show the wireframe and the shaded computer model of the object while figure (13) shows the computer model image taken by an assigned camera in 3D.

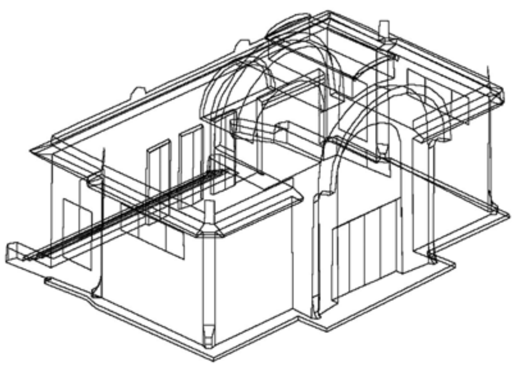

Figure (11) Computer model Wirefram (Hidden and Visible surfaces)

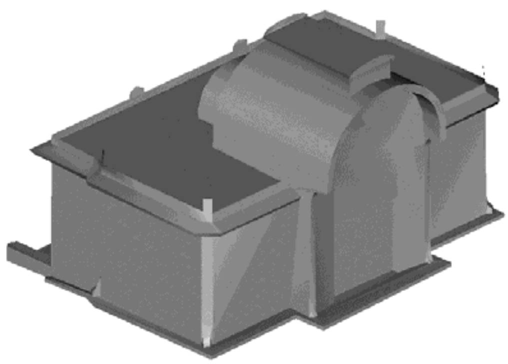

Figure (12) Shaded Computer Model (Visible Surfaces Only)

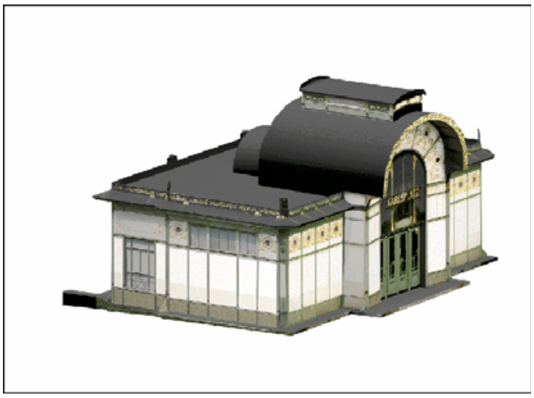

Figure (13) An image for the Computer Model

6.2. Caracalla Thermal Spring (Rome, Italy)

It is a part of ancient “Caracalla Thermal Spring“ in Rome, Italy. This photogrammetric project was originally part of an archaeological investigation of the interesting monument by the Austrian Academy of Science, which has been done in 1988. The rough and weather-beaten surface itself is very different to that of the Karlsplatz object and looks more like a DTM. The nature of the object is very difficult because its surface is irregular and big part of it is an irregular cylinder. Figures (14) & (15) show the object wireframe and shaded computer model while figure (16) shows the computer model with its real shape.

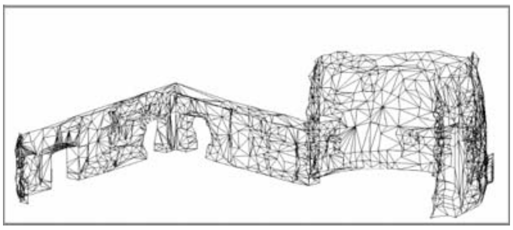

Figure (14) Computer model Wireframe (Hidden and Visible surfaces)

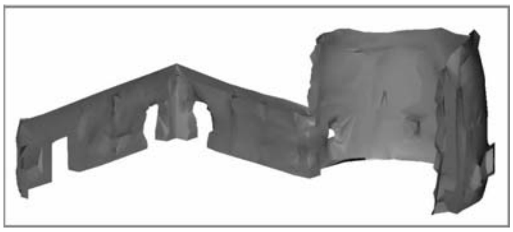

Figure (15) Shaded Computer model (Visible Surfaces Only)

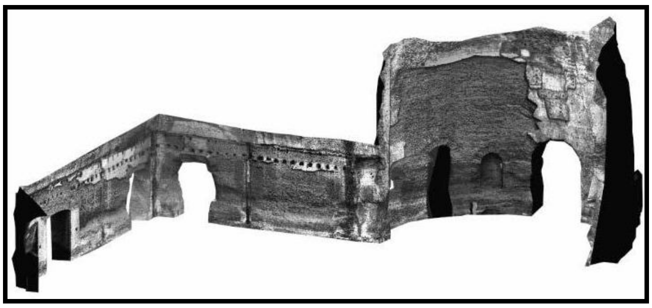

Figure (16) An image for the computer model.

6.3. Statue Head (Luxor, Egypt)

It is one of the old Egyptian statues' head made from marble for one of the old Egyptian Kings, which located in front of the main entrance of the Luxor temple in Luxor city. It is one of the most difficult objects because of the natural of the object's surfaces. It has irregular surfaces, which need large amount of points to be marked to define the face of the statue's head. Figures (17) & (18) show the object wireframe and shaded computer model while figure (19) shows the computer model with its real shape.

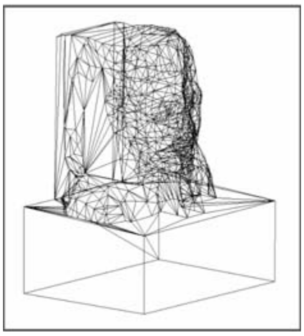

Figure (17) Computer model wireframe (Hidden and Visible surfaces)

Figure (18) Shaded computer model (Visible Surfaces Only)

Figure (19) An image for the computer model

7. CONCLUSION

Creating 3D computer models with their real shapes and dimensions is the best way to document historical buildings and the world's heritage. In these models, the details that appear on the object's surfaces some times need to be measured. Several still images for the computer model can be taken from any desired point of view that shows the interesting details to be measured as far as a camera within 3D Studio can be freely assigned. Only the spatial coordinates of the points used to create the model are available in the database of the model. However, other details on the model's surfaces cannot be directly determined. The main aim of this paper is to introduce a new algorithm for determining the spatial coordinates of these points. The surfaces of the object are considered surfaces of the second order because this type of surface is extensivly used in the objects. In the image plane, parts of some surfaces are invisible because they are located behind each other. The visible parts are divided into plane regions.

The main idea of the new algorithm is to determine the surface to which each region was referenced. To achieve this goal, the basic concepts of the central projection, the computer graphics and the techniques of digital close-range photogrammetry with known camera orientations are used.

This promising new measuring algorithm will enable photogrammetrists to do their job easily and with in the desired accuracy. The advantage of this algorithm is the ability to measure any details laying on the surfaces of computer models.

REFERENCES

Abdelhafiz A., 2009. *Integrating Digital Photogrammetry and Terrestrial Laser Scanning*, Ph.D Dissertation, Nr 23, Braunschweig, ISBN 3-926146-18-4.

Ebrahim M. and Elsonbaty A.,2008. Space Coordinates Of Computer Model's Details Using Digital Close-Range Photogrammetry, *Faculty of Eng. Journal, Al-Azhar University* , Vol. (3)) N. (1), pp. 180-193.

Ebrahim M. A-B., 1998, "Application And Evaluation Of Digital Image Techniques In Close Range photogrammetry", Ph.D. Dissertation at Innsbruck University, Austria.

Elsonbaty A., 2008. Construction of the Convex Hull For a Set of Points in 2D Using The Shooting Method, *Helwan University, Engineering Research Journal*, Vol. 115, pp. P1-P12.

Elsonbaty A., 2008. Displaying Quadratic Surfaces on Computer Screen, *Journal Engineering Science, Assiut University*, Vol. 36, No. 3, pp. 589-601.

Elsonbaty A and Ebrahim M., 2002. A New General Method for Determining 3D Measurements Using Single Image, *Proceedings of the International Conference of Geometry and Graphics ICGG*: Vol. 2, pp 227-231.

Elsonbaty A and Stachel H., 1997. Generating Solids by Sweeping Polyhedra", *Journal of Geometry and Graphics (1)*, 13-21.

Farin G., 1990. Curve and Surfaces for Computer Aided Geometric Design, *Academic Press*, New York.

Forkert G. and Gaisecker Th., 2002. 3D Rekonstruktion von Kultur-guetern mit Laserscanning und Photogrammetrie, *CULTH2 Congress*, MUMOK, Vienna, Jan. 13.

Grimm A., Hanke K, and Ebrahim M., 2001. Documentation and Monitoring of damages of Historical Buildings using a Combination of Different Surveying Techniques, *Faculty of Eng. Journal, Al-Azhar University*.

Jansa J., Studnicka N., Forkert G., Haring A., and Kager H., 2004. Terrestrial Laserscanning and Photogrammetry – Acquisition Techniques Complementing One Another, *Proceedings of the ISPRS XXth Congress, Vol. XXXV, Part B/7*, Istanbul, July 12-23.

Hanke K, Ebrahim M, 1999. The 'Digital Projector' Raytracing as a Tool for Digital Close-Range Photogrammetry, *Elsevier, ISPRS Journal for Photogrammetry & Remote Sensing (54)*, 35-40.

Hanke K, Ebrahim M, 1996. A General Approach For Object Oriented 3d-Mapping In Digital Close Range Restitution, *International Archives of Photogrammetry and Remote Sensing, Vol. XXXI, Part B5, Vienna, p. 215-219.*

Kager, H., 1980. Das interaktive Programmsystem ORIENT im Einsatz, In: *International Archives of Photogrammetry, Hamburg, Vol. XXIII/5, 1980 pp..390-401.*

Kraus K., 1996. *Photogrammetry*, Volum 2: Advanced Methods and Applications, 3rd edition, Buemmler/Bonn.

Hans-Gerd Maas and Uwe Hampel, 2006. Photogrammetric Techniques in Civil Engineering Material Testing And Structure Monitoring, *Photogrammetric Engineering & Remote Sensing Vol. 72, No. 1, January.*

Streilein, A. 1995. Videogrammetry and CAAD for Architectural Restitution of Otto-Wagner-Pavillion in Vienna, *Gruen/Kahmen (Eds), Wichmann Verlag, Heidelberg, pp. 305-314*

Waldhäusl, P. 1991. A test object for architectural photogrammetry: Otto Wagner's Underground station "Karlsplatz" in Vienna, *Proceedings of the XIVth International Symposium of CIPA in Delphi, Oct. 2nd - 5th, Technical Chamber of Greece 1993, pp. 247 - 251.*

Walford Alan, 2010. A New Way to 3D Scan, an article, URL: http://www.photomodeler.com (Last date accessed: Aug. 2010).

Appendix I
Relation between Image and Spatial Coordinates

Let us assume that the coordinates of a point $P(x_P, y_P, z_P)$ in space are known relative to the xyz coordinate-system with the position vector $\mathbf{p}$. The image coordinates of point $P^c(x, y)$ (image of P) need to be calculated.
Let S be the center of projection with position vectors $\mathbf{s}$, and O be the principal point in image plane with position vector $\mathbf{o}$.

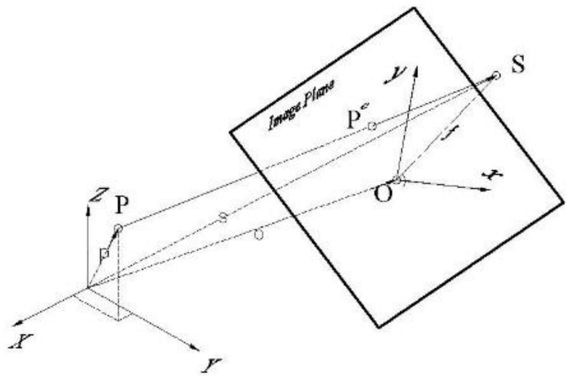

Figure (I-1) Relation between image and spatial coordinates.

The image coordinates of P^c can be calculated from (Ebrahim M. and Elsonbaty A., 2008) as follows:

$$x = \lambda(\mathbf{p}-\mathbf{o})\cdot\mathbf{e}_1$$
$$y = \lambda(\mathbf{p}-\mathbf{o})\cdot\mathbf{e}_2 \qquad \text{(I-1)}$$

where λ is a parameter determined from the following equation:

$$\lambda = \frac{f}{(\mathbf{p}-\mathbf{o})\cdot\mathbf{d}+f} \qquad \text{(I-2)}$$

where $\mathbf{d}$ is the unit vector parallel to $\overline{SO}$ (i.e., the normal on the image plane), f is the focal length, and $\mathbf{e_1}$ and $\mathbf{e}_2$ are unit vectors along the x and y axes in the image plane.

To find the relation between the image coordinates of point $P^c(x,y)$ and the spatial coordinate of $P(x_P, y_P, z_P)$, the above equation can be rewritten in the following form using homogeneous coordinates:

$$x = \frac{x_1}{x_0}, \quad \text{and} \quad y = \frac{x_2}{x_0} \quad \text{with} \quad x_0 = \frac{1}{\lambda} = 1 + \frac{(\mathbf{p}-\mathbf{o})\cdot\mathbf{d}}{f}$$

Hence, $x_1 = (\mathbf{p}-\mathbf{o})\cdot\mathbf{e_1}$ $\quad x_2 = (\mathbf{p}-\mathbf{o})\cdot\mathbf{e_2}$.

Then, the relation between the image coordinates of point $P^c(x_0, x_1, x_2)$ and the space coordinate of $P(x_P, y_P, z_P)$ can be presented in a following matrix form:

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1-\frac{\mathbf{o}\cdot\mathbf{d}}{f} & \frac{d_1}{f} & \frac{d_2}{f} & \frac{d_3}{f} \\ -\mathbf{o}\cdot\mathbf{e_1} & e_{11} & e_{12} & e_{13} \\ -\mathbf{o}\cdot\mathbf{e_2} & e_{21} & e_{22} & e_{23} \end{bmatrix} \begin{bmatrix} 1 \\ x_P \\ y_P \\ z_P \end{bmatrix} \qquad \text{(I-3)}$$

where d_1, d_2, d_3, e_{11}, e_{12}, e_{13} and e_{21}, e_{22}, e_{23} are components of the vectors $d, \mathbf{e_1}, \mathbf{e_2}$ respectively.

CONTACTS

Prof. Dr. Ahmed Ahmed Elsonbaty
Civil Engineering Department
Faculty of Engineering
Qassim University
Qassim
Kingdom of Saudi Aarabia
Asonbaty2@yahoo.com

Ass. Prof. Dr. Mostafa Abdel-Bary Ebrahim
Civil Engineering Department
Faculty of Engineering
Assiut University
Assiut
Egypt.
Mostafa.Ebrahim@Btinternet.com