Investigations to the Calibration of a Numerical Slope Model by Means of Adaptive Kalman-Filtering

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Key words: Landslides, monitoring, slope model, calibration, adaptive Kalman-filtering

SUMMARY

Because of increasing settlement activities of people in mountanious regions and the simultanous appearance of extreme climatic conditions the investigation and alerting of landslides becomes more and more important. Within the last few years a significant rising of disastrous slides could be registered which generated a broad pulic interest and the request for security measures.

In this paper the FWF (Austrian Science Fund) funded project 'KASIP' is presented, which deals with the development of a new type of alarm system based on a calibrated numerical slope model for realistic calculation of failure scenarios. In this context, calibration means the optimal combination of available monitoring data with a numerical model by means of adaptive Kalman-filtering.

The presented study object is the landslide 'Steinlehnen' near Innsbruck (Tyrol, Austria). The first part of the paper is focussed on the determination of geometrical 'surface'-information and includes the description of the monitoring system for the collection of displacement data. The second part is more focussed on investigations to the numerical modelling of the slope by FD- (Finite Difference-) methods and the development of the adaptive Kalman-filter. First filter results are presented and discussed.

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1. MOTIVATION

Because of increasing settlement activities of people in mountainous regions and the simultanous appearance of extreme climatic conditions, the investigation of mass movements becomes more and more important. Within the last few years a significant rising of disastrous landslides could be registered which generated a broad pulic interest and the request for security measures. Following this request, the exploration of early warning systems aims for a raise of security and a restriction of human, economical and environmental damage. This trend is clearly reflected by the EU (European Union) exploratory focusses 'Natural Hazards' and 'Natural Hazard- respectively Disaster-Management'.

For a deeper understanding and explanation of mass movements, static and dynamic numerical models are developed which try to represent the failure mechanisms in a preferably realistic way. Current tools for modelling are given by FE- (Finite Element), FD- (Finite Difference) or DE- (Distinct Element-) codes (e.g. ITASCA, 2010). The initial values for the physical model parameters (e.g. strength parameters like friction and cohesion) are usually derived from geological and geophysical advance informations, this means geological maps and/or in situ investigations like seismic measurements. The further adaptation of the numerical models to the geometrical monitoring data of alert systems (e.g. tacheometer and levelling data, GPS measurements, etc.) is then normally realised by 'trial and error'-methods, which means varying the parameters in a small range around the current working point until calculated and monitored displacement data are fitting to each other (e.g. MEIER et al., 2008). Until now, a real model calibration (including a statistical evaluation of the results) in terms of adjustment methods is not state of the art.

One aim of the FWF (Austrian Science Fund) funded project 'KASIP' (Knowledge-based Alarm System with Identified Deformation Predictor) is the investigation of new methods for the calibration of a numerical slope model. The basic idea for the calibration process is to use adaptive Kalman-filtering techniques (e.g. GELB et al., 1974 and HEUNECKE, 1995), which principally enable the optimal estimation of the system state (e.g. the kinematic state and relevant material parameters) of the slope. Using Kalman-filtering, a significant model improvement is expected which is the precondition for the realistic prediction and simulation of the slope properties within the framework of a new type of alert system (KASIP, 2009 and SCHMALZ et al., 2010).

2. STUDY OBJECT

The selected study object in KASIP is the slope 'Steinlehnen' in Northern Tyrol (see Figure 1) which performs a significant mass movement since 2003 (ZANGERL et al., 2007 and

SCHMALZ et al., 2010). As the slope is situated close to buildings and a national road, a periodic monitoring with a tacheometer system from the opposite slope is performed in discrete points. These points are signalised by prisms. The monitoring has been carried out by 'alpS – Centre for Natural Hazard and Risk Management' and since one year by the 'Geodetic Institute', TU Darmstadt (see Figures 2 and 3).

Figure 1: Study site 'Steinlehnen'

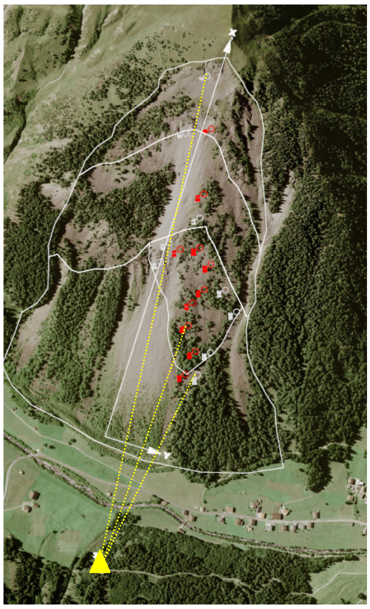

Figure 2: Monitoring design

The measurement rate varies between Δt = 1 month and 1 year. After an acceleration phase in spring 2004, the slope is currently moving with about 0.25 m / year down into the valley (ZANGERL et al., 2007). As can be seen in Figure 2, the monitoring points are situated in the periphery of the slope. Because of possible rockfalls, the adaptation of prisms in the center is too dangerous. A measuring campaign with a terrestrial radar system (IBIS, see IDS, 2010) is planned for spring 2010 to close this data gap.

Figure 3: Tacheometer station at the opposite slope

A dynamic numerical 3D-model of the slope is currently developed by the engineering geologists of TU Vienna (MAIR AM TINKHOF et al., 2009) and shall be calibrated by adaptive Kalman-filtering using the collected monitoring data. The numerical model is realised with the software FLAC3D (Fast Lagrangian Analysis of Continua in three Dimensions, see ITASCA, 2010) which is based on the Finite Difference method. The software enables the calculation of continuum models on discrete 3D-grids using different elastic and plastic material models. In Figure 4 the planned association between real measurements and numerical calculations is shown. The model contains about 100.000 grid points.

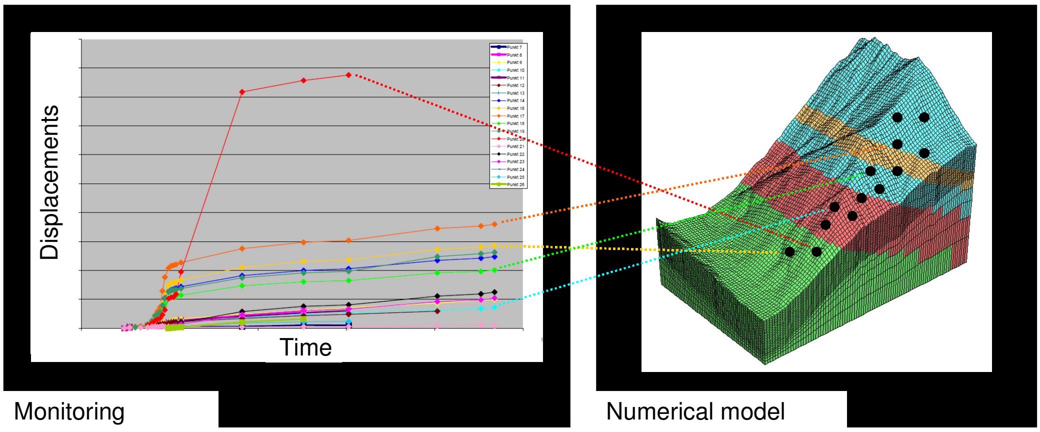

Figure 4: Association between monitoring data and the numerical slope model

3. INVESTIGATIONS FOR MODEL CALIBRATION

3.1 Numerical modelling of a simplified test slope

The investigations for the feasibility of the FLAC3D-model calibration are performed in simulations with a simplified test slope consisting of only 704 discrete grid points (Figure 5).

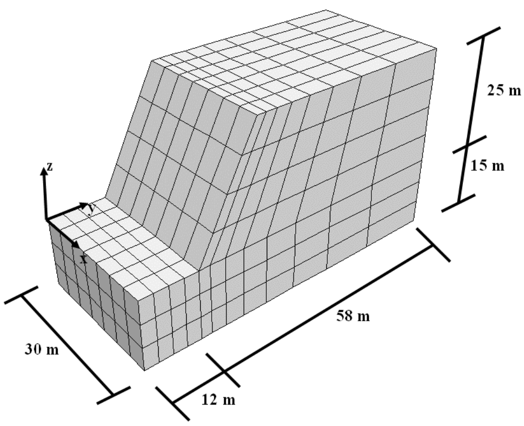

Figure 5: Simplified numerical slope model

The main reasons for this are the a priori knowledge of the model parameters which enables the significant evaluation of the filter results and the reduction of calculation time. Even in the reduced model, the calculation of one filter step still occurs within the range of several hours, as will be shown in Section 3.3.

Based on the applied forces (equations of motion) affecting each single point of the finite difference grid, displacements and velocities are calculated. Following that, FLAC3D calculates the strain rates for all grid zones. Calculations end, if the 'unbalanced force' has reached a certain value (ROTH, 1999). Here, the unbalanced force describes the unbalance of forces affecting the entire system (e.g. the finite difference grid).

The simplified numerical slope model has a depth of 40m, a width of 30m and a length of 70m. Boundary conditions are defined in such a way, that the model is fixed at z = -15m for the xy-layer in x-, y- and z-direction, at x = 0m and x = 30m for the yz-layers in x-direction and for y = 0m and y = 70m for the xz-layers in y-direction

The behaviour of the rock mass is simulated using a Mohr-Coulomb material model. Here, the entire slope model is considered to be homogenous. The chosen values for the density ρ, bulk modulus E and Poisson's ratio ν can be found in Table 1. Using this configuration, the limited state of equilibrium is calculated by reduction of the strength parameters cohesion c and angle of internal friction φ. This calculation can be performed automatically in FLAC3D. The boundary values c_B and φ_B describe material properties, where the limited state of equilibrium is reached.

Table 1: Boundary values c_B and φ_B for the stability of the test slope

ρ [kg/m^3]	E [MN/m^2]	ν [ ]	c_B [KN/m^2]	φ_B [°]
2500	257	0.29	26.76	35.15

In order to investigate the adaptation of the FLAC3D-model by adaptive Kalman-filtering techniques, slope deformations are simulated by systematic reduction of the strength parameters below the boundary state of equilibrium (reference parameterisation for the deformation process). The obtained synthetic deformation datasets serve as 'observations' and input for the Kalman-filter.

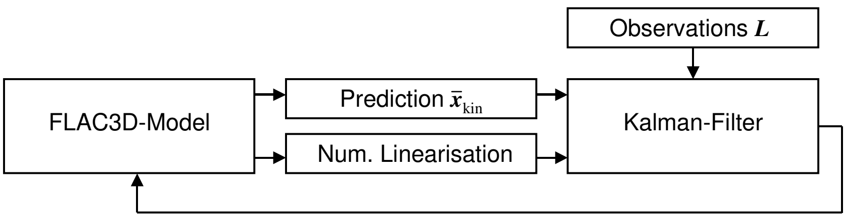

Figure 6: Process of Kalman-filtering

The Kalman-filter algorithm shall be used to fit the slope model with the observations. Dependent upon current load and estimation of the state vector in the previous filterstep, the prediction is directly calculated using the FLAC3D-model. As the model equations are not analytical accessible, the error propagation requires a numerical linearisation of the FLAC3D-model in each filterstep (see Figure 6).

3.2 Adaptive Kalman-filter for a static model

Considering a static model, the output signal is only dependent on the physical load at certain timesteps, where the whole system is in balanced condition. Here, load means the strength parameters (cohesion and angle of internal friction) and gravitational force. Since the gravitational force is assumed to be non-stochastic and other loads (e.g. ground water table) are not integrated in the simplified numerical model, the Kalman-filter is tested with one strength parameter 'triggering' the deformation and the other parameter being estimated. This is shown exemplarily with the angle of internal friction acting as 'observed' trigger and the cohesion being the 'a priori unknown' estimated value.

The current system state $\boldsymbol{x}_{\mathrm{erw},k}$ is depicted as

$$\begin{aligned}\boldsymbol{x}_{\mathrm{erw},k} &= (\underbrace{x_1,\ldots,x_{704} \mid y_1,\ldots,y_{704} \mid z_1,\ldots,z_{704}}_{\mathrm{kin}} \mid \underbrace{c}_{\mathrm{p}})_k^{\mathrm{T}} \\ &= (\boldsymbol{x}_{\mathrm{kin},k}^{\mathrm{T}} \mid \boldsymbol{x}_{\mathrm{p},k}^{\mathrm{T}})\end{aligned} \quad , \tag{1}$$

where $\boldsymbol{x}_{\mathrm{kin}}$ describes the kinematic part and $\boldsymbol{x}_{\mathrm{p}}$ the adaptive part of the state vector. The error propagation for the prediction is considered as

$$\boldsymbol{\varepsilon}_{\bar{\boldsymbol{x}}_{\mathrm{erw}},k+1} = \begin{pmatrix} \boldsymbol{\varepsilon}_{\bar{\boldsymbol{x}}_{\mathrm{kin}},k+1} \\ \boldsymbol{\varepsilon}_{\bar{\boldsymbol{x}}_{\mathrm{p}},k+1} \end{pmatrix} = \underbrace{\left(\begin{array}{c|c} \boldsymbol{T}_{k+1,k} & \boldsymbol{T}_{\mathrm{p},k+1,k} \\ \hline \boldsymbol{O} & \boldsymbol{E} \end{array}\right)}_{\boldsymbol{T}_{\mathrm{erw},k+1,k}} \begin{pmatrix} \boldsymbol{\varepsilon}_{\hat{\boldsymbol{x}}_{\mathrm{kin}},k} \\ \boldsymbol{\varepsilon}_{\hat{\boldsymbol{x}}_{\mathrm{p}},k} \end{pmatrix} + \underbrace{\begin{pmatrix} \boldsymbol{B}_{k+1,k} \\ \hline \boldsymbol{O} \end{pmatrix}}_{\boldsymbol{B}_{\mathrm{erw},k+1,k}} \boldsymbol{\varepsilon}_{\boldsymbol{u},k} + \underbrace{\left(\begin{array}{c|c} \boldsymbol{S}_{k+1,k} & \boldsymbol{S}_{\mathrm{p},k+1,k} \\ \hline \boldsymbol{O} & \boldsymbol{E} \end{array}\right)}_{\boldsymbol{S}_{\mathrm{erw},k+1,k}} \begin{pmatrix} \boldsymbol{w}_k \\ \boldsymbol{w}_{\mathrm{p},k} \end{pmatrix} \tag{2}$$

with $\boldsymbol{T}_{k+1,k}$ as an identity matrix $\boldsymbol{E}$ and $\boldsymbol{T}_{\mathrm{p},k+1,k}$ and $\boldsymbol{B}_{k+1,k}$

$$\boldsymbol{T}_{\mathrm{p},k+1,k} = \begin{pmatrix} \dfrac{\Delta\bar{x}_1}{\Delta c} \\ \vdots \\ \dfrac{\Delta\bar{z}_{704}}{\Delta c} \end{pmatrix}_{k+1,k} \quad ; \quad \boldsymbol{B}_{k+1,k} = \begin{pmatrix} \dfrac{\Delta\bar{x}_1}{\Delta\varphi} \\ \vdots \\ \dfrac{\Delta\bar{z}_{704}}{\Delta\varphi} \end{pmatrix}_{k+1,k} \tag{3}$$

considering the adaptive part (cohesion c) and the trigger (angle of internal friction φ). $\boldsymbol{T}_{\mathrm{erw},k+1,k}$ is the transition matrix and $\boldsymbol{B}_{\mathrm{erw},k+1,k}$ the influence matrix of the correcting variable.

The system noise is represented by the stochastic disturbance velocities $\boldsymbol{w}_k$ ($\mathrm{E}\{\boldsymbol{w}_k\} = \boldsymbol{o}$). Here, the disturbance input matrix $\boldsymbol{S}_{k+1,k}$ can be written as

$$S_{k+1,k} = \begin{pmatrix} \Delta t & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & \Delta t \end{pmatrix} \quad \text{and} \quad \boldsymbol{w}_k = (wv_{x1} \ \cdots \ wv_{z704})_k^{\mathrm{T}} \,. \tag{4}$$

The stochastic disturbance $\boldsymbol{w}_{p.k}$ ($\mathrm{E}\{\boldsymbol{w}_{p,k}\} = \boldsymbol{o}$) of the adaptive part is considered in the random walk process

$$\underbrace{(c_{k+1})}_{\boldsymbol{x}_{p,k+1}} = \underbrace{(c_k)}_{\boldsymbol{x}_{p,k}} + \underbrace{(w_{c,k})}_{\boldsymbol{w}_{p,k}}. \tag{5}$$

As already mentioned in Section 3.1, synthetic deformation datasets are generated by the reduction of the strength parameters below the state of equilibrium. In the static case, the approach can be described as follows. For every new dataset, the angle of internal friction is stepwise reduced while the cohesion is defined as a fixed (but reduced compared to c_B) value. Each calculation of static deformation is performed from the same initial balanced state of the slope. The defined cohesion c_{ref} is the reference value to be estimated after the filtering process.

Table 2: Configuration for the static model

ρ [kg/m^3]	E [MN/m^2]	ν [ ]	c_{ref} [KN/m^2]	$\varphi_{trig,\,k}$ [°] mit k = 1, 2, ...
2500	257	0.29	26.6	35.4
2500	257	0.29	26.6	35.3
2500	257	0.29	26.6	35.25
2500	257	0.29	26.6	35.21

The input configuration for the generation of the synthetic deformation datasets can be found in Table 2. One might argue here, that initial values for $\varphi_{trig,\,k}$ are chosen that are greater than φ_B. The reason is, that slight deformation behaviour within the range of several millimetres up to a few centimetres is already generated using these values. Here, elastic material behaviour is simulated. Using lower values for $\varphi_{trig,\,k}$ would cause plastic material behaviour, i. e. deformation up to several meters.

The error equation for the observations $\boldsymbol{L}_{k+1} = (x_1 \ \ldots \ z_{704})^{\mathrm{T}}{}_{k+1}$ is specified as

$$\boldsymbol{\varepsilon}_{L,k+1} = \boldsymbol{L}_{k+1} - \boldsymbol{A}_{k+1,k} \ \boldsymbol{x}_{erw,k+1} \tag{6}$$

where $\boldsymbol{A}_{k+1,k}$ is the design matrix with

$$\boldsymbol{A}_{k+1} = \left(\begin{array}{ccc|c} \overset{x_1}{1} & \cdots & \overset{z_{704}}{\boldsymbol{O}} & \overset{c}{0} \\ & \ddots & & \vdots \\ \boldsymbol{O} & & 1 & 0 \end{array}\right). \tag{7}$$

Figure 7 shows the results for the estimation of the cohesion over several epochs starting with different initial values. As can be seen, the reference value c_{ref} is reached within the first two or three filter steps. The filter results are independent from a variation of the initial values c_0.

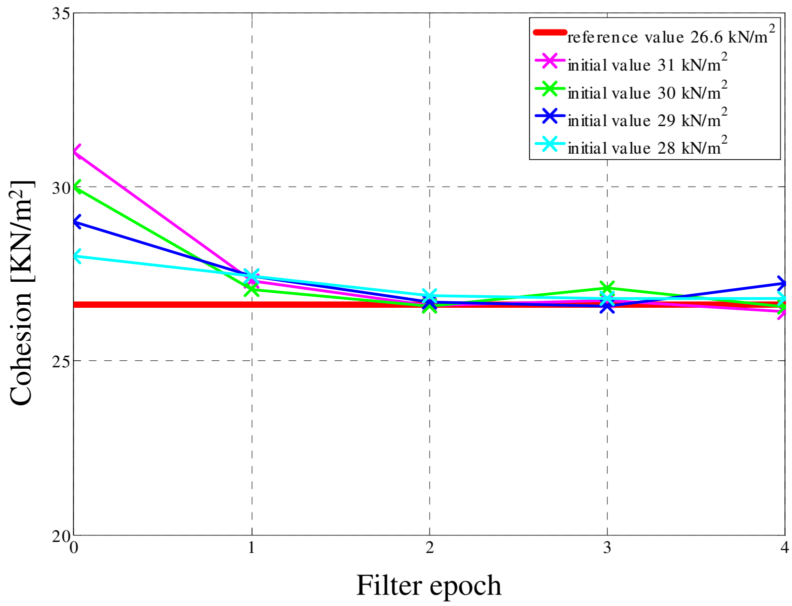

Figure 7: Estimation of the cohesion for a static model approach

3.3 Adaptive Kalman-filter of a dynamic model

In contrast to the static model approach, dynamic deformation processes are strongly time-dependent. In the following experiment, the gravitational force is the only occurring load. As described in Section 3.2, the gravitational force is non-stochastic and consequently not considered in the error propagation.

FLAC3D allows the generation of dynamic synthetic datasets and uses its own time unit called 'steps'. Calculations end, if the unbalanced force between the grid points reaches a predefined minimum (ROTH, 1999). Starting with defined values for the material parameters, synthetic datasets are created every 500 steps, simulating a slight slope deformation. Again, the Kalman-filtering process shall exemplarily be shown with the cohesion c to be estimated, while the angle of internal friction φ is kept as a fixed and known value. The input configuration for the generation of the synthetic deformation datasets is shown in Table 3.

Table 3: Configuration for the dynamic model

ρ [kg/m³]	E [MN/m²]	ν [ ]	c_{ref} [KN/m²]	φ_{ref} [°]
2500	257	0.29	26.6	35.15

For the dynamic model approach, $\boldsymbol{T}_{k+1,k}$ (see equation (2)) also includes the kinematic part of the state vector. No defective trigger is included. This yields to

$$\boldsymbol{T}_{k+1,k} = \begin{pmatrix} \frac{\Delta\bar{x}_1}{\Delta x_1} & \cdots & \frac{\Delta\bar{x}_1}{\Delta z_{704}} \\ \vdots & \ddots & \vdots \\ \frac{\Delta\bar{z}_{704}}{\Delta x_1} & \cdots & \frac{\Delta\bar{z}_{704}}{\Delta z_{704}} \end{pmatrix}_{k+1,k} \quad \text{and} \quad \boldsymbol{B}_{k+1,k} = \boldsymbol{O}. \tag{8}$$

As the Kalman-filter must start in each filter step from the same initial balanced state, the velocities and accelerations can be considered to be zero and do not contribute to the error propagation. The reason for this unusual filter strategy is, that FLAC3D cannot calculate deformations in a recursive way, i.e. introducing new estimated (deformed) grids in the calculation process.

Table 4 shows the increasing computing time for the determination of $\boldsymbol{T}_{k+1,k}$ depending on the FLAC3D step rate. Calculations were performed using an ordinary personal computer (Intel Core2 Quad CPU Q6600 @ 2.40 GHz, 4 GB RAM). Considering equation (8), the computing time might be very challenging if applied to a much larger grid.

Table 4: Computing time for the determination of $\boldsymbol{T}_{k+1,k}$

Epoch	1	2	3	4	5
Steps	500	1000	1500	2000	2500
Time (hrs:min)	01:50	02:26	03:06	03:25	04:01

Figure 8 shows the results for the estimation of the cohesion c over several epochs starting with different initial values. The reference value c_{ref} is almost reached after two filter steps.

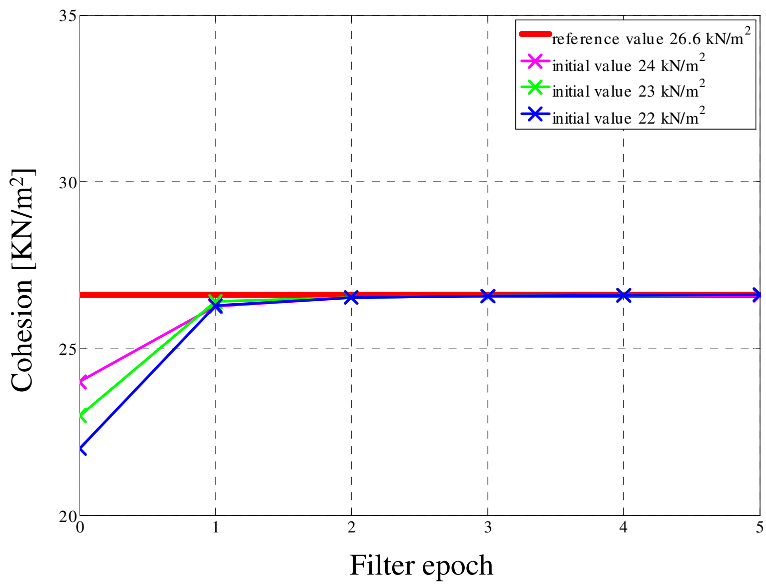

Figure 8: Estimation of the cohesion for a dynamic model approach

4. CONCLUSIONS

Nowadays, investigations of mass movements are often done by the use of numerical slope models. The adaption of those models is then typically realized by trial and error-methods. A stochastic calibration of numerical slope models by the use of adaptive Kalman-filtering has never been done before. First results applied to a simplified numerical slope model show, that the estimation of strength parameters is possible.

In the next step, adaptive Kalman-filtering techniques shall be used to calibrate the finite-difference model of the mass movement 'Steinlehnen' (Tyrol, Austria). Here, the grid size will exceed 100.000 points. The implementation of possible 'triggers' causing failure mechanisms, computing time and memory overflow are currently the challenging investigation topics.

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