Evaluation of the Effects of the Observation Weight Matrix on the Ambiguity Resolution in LAMBDA Method

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SUMMARY

Carrier beat phase measurements of GPS are routinely used for precise positioning applications. However, before any position can be computed, the carrier ambiguity which is an inherent part of these types of observations should be correctly resolved. Otherwise, the computed positions are biased and their corresponding accuracies are drastically decreased. Ambiguity resolution is still a challenging problem in precise positioning applications. Finding a fast and reliable method for ambiguity resolution is notwithstanding an open area of research.

Precise positioning computations using carrier beat phase observations usually involve three steps: 1. resolving ambiguities as float numbers, 2. assigning integer numbers to the resolved ambiguities (fixing ambiguities), and 3. testing the fixed ambiguities and computing accurate positions. Researchers have introduced many different methods to resolve the ambiguities. However, LAMBDA (Least square AMBiguity Decorrelation Adjustment) is one of the most powerful ones used so far.

This paper discusses the effects of observation weight matrix on ambiguity resolution using LAMBDA method. Numerical results show that observation weight matrix has a direct effect on the accuracy of resolved float ambiguities. Moreover, in the process of fixing ambiguities using LAMBDA method, observation weight matrix determines the dimension of search space but it has no direct effect on fixing the ambiguities. Last but not least, it is shown that the result of adjustment using fixed ambiguities has not been much affected with the quality of the observation weight matrix unless it is unreasonably wrong.

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1. INTRODUCTION

Nowadays, GPS is used for many precise positioning applications. To take advantage of GPS for precise positioning one should use carrier beat phase observations. The inherent part of these measurements is the ambiguity which should be resolved before the observations can be used for computation of the coordinates. The correct solution of ambiguities needs long and reliable observation set. Many methods have been developed to resolve the ambiguities fast and reliable. One of these methods is LAMBDA (Least square AMBiguity Decorrelation Adjustment) which takes advantage of decorrelating the ambiguities and solves them with a higher speed without reducing the reliability.

The ambiguity resolution process starts with estimating the ambiguities as float numbers. Various algorithms have been developed to find out the integer ambiguities from the float solution. Then the integer solution is tested. Different tests are used to increase the reliability of ambiguity resolution. If the test is passed, the integer ambiguities are used in the observation equations as known values. Otherwise, one should either increase the length of observation, or somehow reduce the errors or biases and again solve for ambiguities.

In this paper, carrier beat phase observation is reviewed. Then the ambiguity resolution process and some of the available techniques including LAMBDA are briefly reviewed. Finally, the implemented algorithm as well as the results using different observation weight matrix are discussed.

2. CARRIER PHASE

Global Positioning System (GPS) provides the position of users using signals of four or more satellites. The system was primarily designed of 24 satellites in 6 different orbits, but at present there are 30 satellites in the orbits (http://www.navcen.uscg.gov/). The satellites transmit two C/A and P codes modulated on two L1 and L2 carriers (Bao, 2004). The carrier beat phase observations are used for precise positioning applications. In order to determine the distance between the receiver and satellite using carrier phase observations one should measure the number of carrier cycles. However, the receivers are only able to measure the fraction of carrier cycle when the signals are locked. Therefore, the integer cycles between the receiver and satellite which is called ambiguity should be determined. Equation (1) shows this in mathematical language (Wells et al., 1987):

$$\Phi_{total} = Frac(\Phi) + Int(\Phi, t, t_0) + N(t_0) \quad (1)$$

where $Frac(\Phi)$ is the fraction of carrier cycle which is measured in the receiver at the time t_0 when the signal is locked in the receiver, $Int(\Phi, t, t_0)$ is the integer cycles which the receiver has counted between the time when the signal is locked and time t, $N(t_0)$ is the integer cycles of carrier, and Φ_{total} is the total carrier phase between the satellite and receiver. The carrier beat phase observation equation is as following (El-Rabbany, 2002):

$$\Phi = \rho + d\rho + c(dT - dt) - d_{ion} + d_{trop} + \lambda N + \varepsilon$$
(2)

where Φ is the measured distance by the receiver, ρ is the geometric distance between the satellite and receiver, $d\rho$ is the orbital error, $c(dT - dt)$ is satellite's and receiver's clock errors, d_{trop} and d_{ion} are the tropospheric and ionospheric errors, respectively, N is the ambiguity, λ is the wavelength, and ε is the observation noise.

Differential or relative positioning is used to reduce or eliminate the errors. Differencing between receivers eliminates the satellite's clock error while differencing between satellites eliminates the receiver's clock error. Generally speaking, orbital and atmospheric errors are reduced in differential positioning. Double differencing is used for precise positioning applications (Teunissen, 1998). Double differencing eliminates the satellite and receiver clock errors while reduces other ones. The double differencing observation equation is as following (El-Rabbany, 2002):

$$\Delta\nabla\Phi = \Delta\nabla\rho + \Delta\nabla d\,\rho + c(\Delta\nabla dT - \Delta\nabla dt) - \Delta\nabla d_{ion} + \Delta\nabla d_{trop} + \Delta\nabla N + \Delta\nabla\varepsilon$$
(3)

Where "$\Delta\nabla$" is the notation for double differencing.

3. AMBIGUITY RESOLUTION

To reach a position with the accuracy of the order of centimeter in static mode or 10 centimeter in kinematic mode it is necessary to solve the ambiguity. Finding a fast and reliable method for ambiguity resolution is still an on going research topic. Generally speaking, ambiguity resolution process consists of the following three steps (Liu, 2005):

1. Finding the float solution. In this step, the float ambiguities are estimated using the measurements. This solution is good for applications which need an accuracy of order of 50 cm or so. Least Squares, Kalman filtering, and combination of code with carrier phase can be used in this step (Chen, 1994).
2. Searching for integer ambiguities. In this step, a search space is set based on the variances of the estimated float ambiguities. Finding integer ambiguities among float ones is usually done based on minimum distance (Euclidian norm).

3. Testing the integer ambiguities to increase the reliability of the solution. The test shows whether the integer solution is within an acceptable range of reliability or not.

3.1 Float Solution of the Ambiguities

In this paper, Least Squares is used for the float solution. Least Squares is a linear estimator and for nonlinear models one should linearize it (Vanicek, 1986). The linear model can be written as:

$$l = Ax + w \tag{4}$$

Where x is the unknown vector, A is the design matrix, l is the observation vector and w is the misclosure vector. The unknown vector in double differencing is:

$$x = [x_{rov}\, y_{rov}\, z_{rov}\, \Delta\nabla N_{rov-ref}^{base-2}\, \Delta\nabla N_{rov-ref}^{base-3} \cdots \Delta\nabla N_{rov-ref}^{base-n}] \tag{5}$$

where the first three unknowns are the coordinates of the rover and the rest are the differences of the ambiguities in differencing between satellites and receivers. The design matrix can be written as:

$$A = \frac{\partial f}{\partial x}\Big|_{x_0} = \begin{bmatrix} \frac{\partial\Delta\nabla\rho^{base-2}}{\partial x} & \frac{\partial\Delta\nabla\rho^{base-2}}{\partial y} & \frac{\partial\Delta\nabla\rho^{base-2}}{\partial z} & 1 & -1 & 0 \ldots & 0 \\ \frac{\partial\Delta\nabla\rho^{base-3}}{\partial x} & \frac{\partial\Delta\nabla\rho^{base-3}}{\partial y} & \frac{\partial\Delta\nabla\rho^{base-3}}{\partial z} & 1 & 0 & -1 \ldots & 0 \\ \vdots & & & & & & \\ \frac{\partial\Delta\nabla\rho^{base-n}}{\partial x} & \frac{\partial\Delta\nabla\rho^{base-n}}{\partial y} & \frac{\partial\Delta\nabla\rho^{base-n}}{\partial z} & 1 & 0 & 0 \ldots & -1 \end{bmatrix} \tag{6}$$

where

$$\frac{\partial\Delta\nabla\rho^{base-i}}{\partial x} = \frac{x_{rov} - x^i}{\rho_{rov}^i} - \frac{x_{rov} - x^{base}}{\rho_{rov}^{base}}$$
$$\frac{\partial\Delta\nabla\rho^{base-i}}{\partial y} = \frac{y_{rov} - y^i}{\rho_{rov}^i} - \frac{y_{rov} - y^{base}}{\rho_{rov}^{base}} \tag{7}$$
$$\frac{\partial\Delta\nabla\rho^{base-i}}{\partial z} = \frac{z_{rov} - z^i}{\rho_{rov}^i} - \frac{z_{rov} - z^{base}}{\rho_{rov}^{base}}$$

and ρ_{rov}^i is the geometric distance between the rover and satellite i, $(x_{rov}, y_{rov}, z_{rov})$ are the coordinates of the rover, and (x^i, y^i, z^i) are the coordinates of the satellite i. Moreover, the observation vector is as following:

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$$\Delta\nabla\Phi = [(\Phi^2_{rover} - \Phi^{base}_{rover})_1 - (\Phi^2_{ref} - \Phi^{base}_{ref})_1 \cdots (\Phi^n_{rover} - \Phi^{base}_{rover})_1 - (\Phi^n_{ref} - \Phi^{base}_{ref})_1 \cdots$$
$$(\Phi^2_{rover} - \Phi^{base}_{rover})_m - (\Phi^2_{ref} - \Phi^{base}_{ref})_m \cdots (\Phi^n_{rover} - \Phi^{base}_{rover})_m - (\Phi^n_{ref} - \Phi^{base}_{ref})_m\,]$$
(8)

where $(\Phi^i_{ref})_m$ is the observed phase of satellite i in the reference receiver at epoch m, $(\Phi^i_{rov})_m$ is the observed phase of satellite i in the rover receiver at epoch m. The misclosure vector is also written as:

$$w = f(x_0, l)\,.$$
(9)

The corrections to the approximate value of the unknowns, the estimates unknowns, and the variance-covariance of the estimates unknowns are computed using the following equations, respectively:

$$\hat{\delta} = -N^{-1}u = -(A^T C_l^{-1} A)^{-1} A^T C_l^{-1} w$$
(10)
$$\hat{x} = x_0 + \hat{\delta}$$
(11)
$$C_{\hat{x}} = N^{-1} = (A^T C_l^{-1} A)^{-1}$$
(12)

where C_l is the variance-covariance matrix of the observation, $C_{\hat{x}}$ is the variance-covariance matrix of the estimated unknowns and x_0 is the first approximation for the unknowns. The sign "^" shows the estimated values. Since the mathematical model of double differencing is nonlinear, the Least Squares solution is iterated until the solution converges to zero.

3.2 Integer Solution of the Ambiguities

The geometrical distance (Euclidian norm) between integer and float solutions is used to find out the integer solution. In other words (Teunissen, 2000):

$$\min_a((\hat{a} - a)^T Q_{\hat{a}}^{-1} (\hat{a} - a))$$
(13)

Where $\hat{a}$ is the estimated float solution, a is the possible integer solution, and $Q_{\hat{a}}$ is the variance-covariance of the estimated ambiguities. Assuming that the ambiguities have normal distribution, the following relation has χ^2 distribution (Teunissen, 2000):

$$(\hat{a}-a)^T Q_{\hat{a}}^{-1}(\hat{a}-a) \le \chi^2 .$$
(14)

This is nothing else but the equation for a multidimensional ellipsoid which determines the search space. The values of the ambiguities determine the center of the multidimensional ellipsoid and their variance-covariance matrix defines the size of the search space. As the correlation between the float ambiguities is high, one can not easily find the integer solution. LAMBDA uses conditional Least Squares and estimates the ambiguity based on the already estimated ambiguities.

Equation (13) can be rewritten as following (Teunissen, 2000):

$$\min_{a_1,\cdots,a_n \in Z} \sum_{i=1}^{m} \frac{(a_{i|i-1}-a_i)^2}{\delta^2_{\hat{a}_{i|i-1}}}$$
(15)

where $a_{i|i-1}$ is the estimated ith ambiguity based on the previous $i-1$ estimated integer ambiguity,, and $\delta^2_{i|i-1}$ is the estimated variance of ith ambiguity based on the previous $i-1$ estimated integer ambiguity. Therefore, the search space is as following (Teunissen, 2000):

$$\begin{cases} (\hat{a}_1-a_1)^2 \le \delta^2_{\hat{a}_1}\chi^2 \\ (\hat{a}_{2|1}-a_2)^2 \le \delta^2_{\hat{a}_{2|1}}\left[\chi^2 - \dfrac{(\hat{a}_1-a_1)^2}{\delta^2_{\hat{a}_1}}\right] \\ \vdots \\ (\hat{a}_{m|m-1}-a_m)^2 \le \delta^2_{\hat{a}_{m|m-1}}\left[\chi^2 - \displaystyle\sum_{j=1}^{m-1}\dfrac{(\hat{a}_{j|j-1}-a_j)^2}{\delta^2_{\hat{a}_j}}\right] \end{cases}$$
(16)

To increase the efficiency of the algorithm and reduce the number of candidates for the integer ambiguities in the search space without decreasing the reliability one can use the Z matrix as following to decorrelate the ambiguities (Teunissen, 1999):

$$z = Z^T a$$
(17)
$$\hat{z} = Z^T \hat{a}$$
(18)
$$Q_{\hat{z}} = Z^T Q_{\hat{a}} Z$$
(19)

where z is the transformed integer ambiguities vector, $\hat{z}$ is the transformed float ambiguities vector, and $Q_{\hat{z}}$ is the variance-covariance matrix of the transformed float ambiguities. Using the above mentioned transformation one can rewrite the condition (13) as following:

$$\min_{z}((\hat{z}-z)^{T} Q_{\hat{z}}^{-1} (\hat{z}-z)) \cdot$$
(20)

In other words, Z transforms the search space to a new one in which the ambiguities are not correlated.

3.3 Test

Test is used to select the correct integer ambiguities among all of the possible integer ambiguities in the search space. The pessimistic tests increase the time for ambiguity resolution and on the other hand, optimistic tests reduce the reliability (Weisenburger, 1997). Finding an efficient test is still an open research subject. One of the most efficient ones which is used in this paper is the ratio test. This test does not have any special confidence level and has the following form (Verhagen, 2004):

$$\frac{\Omega_2}{\Omega_1} < threshold$$
(21)

where Ω_1 is the first shortest distance between the float and integer solution, Ω_2 is the next shortest one.

4. IMPLEMENTATION

For the purpose of experimentation two Leica 1230 dual frequency GPS receivers are used. The measurements to 6 satellites are collected for 10 minutes at the rate of 1 second in Tehran (Iran) at 10 AM local time. To avoid errors a two-meter baseline is used. In this paper, Least Squares is used for the float solution, LAMBDA is used for integer solution, the ratio test is used for testing the integer solution and again Least Squares is used for solving the unknown coordinates. The algorithm is implemented in MatLab environment and the results are checked with GeoGenius 2000 and GPSurvey softwares.

The purpose of this paper is to check the effect of observation weight matrix on the ambiguity resolution. At the first step, the weight matrix is considered as the identity matrix. In other words, it is assumed that all of the observations have similar weight. In this case, the accuracy of the positioning using float solution is 165 mm. After fixing the ambiguities using LAMBDA method, the positioning accuracy increases to 4 mm. The integer solution is correct and the ratio test is passed with a threshold of 2.

At the second step, the weight of observations is considered as proportional to the altitude of the corresponding satellite. In this case, the accuracy of positioning using float solution is 130 mm. After fixing the ambiguities, the positioning accuracy increases to 4 mm. Similar to the previous case, the integer solution is correct and the ratio test is passed with a threshold of 2.

At the third step, the weight of observations is considered as proportional to different powers of the altitude of the corresponding satellite. At this step, the effect of unreasonable weight matrix is studied. Table (1) summarizes the results.

Table 1: Effect of Weight Matrix on Ambiguity Resolution

Weight Matrix	Float Solution (mm)	Integer Solution (mm)	Test Result	Ambiguity Resolution
$P = I$	165	4	Pass	Correct
$P = a_{sat}$	130	4	Pass	Correct
$P = a_{sat}^2$	101	4	Pass	Correct
$P = a_{sat}^3$	80	4	Pass	Correct
$P = a_{sat}^4$	64	4	Fail	Correct
$P = a_{sat}^5$	54	5	Fail	Correct
$P = a_{sat}^6$	46	201	Fail	Incorrect
$P = a_{sat}^7$	38	203	Fail	Incorrect
$P = a_{sat}^8$	29	205	Fail	Incorrect
$P = a_{sat}^9$	18	208	Fail	Incorrect
$P = a_{sat}^{10}$	1	490	Fail	Incorrect
$P = a_{sat}^{11}$	18	486	Fail	Incorrect
$P = a_{sat}^{12}$	38	479	Fail	Incorrect
$P = a_{sat}^{13}$	53	471	Fail	Incorrect
$P = a_{sat}^{14}$	63	460	Fail	Incorrect
$P = a_{sat}^{15}$	68	450	Fail	Incorrect
$P = a_{sat}^{30}$	56	421	Fail	Incorrect
$P = a_{sat}^2 + a_{Base\_sat}^2$	144	4	Pass	Correct
$P = \sqrt{a_{sat}^2 + a_{Base\_sat}^2}$	154	4	Pass	Correct

5. CONCLUSIONS

The result of this experimentation shows that the observation weight matrix has a direct effect on the accuracy of the positioning using float solution but has no effect on the positioning accuracy using integer solution. Moreover, unless the observation weight matrix is unreasonably selected, the integer solution is correct. Last but not least, it is seen that the ratio test is an efficient one as it fails when the integer solution is wrong and it passes when the solution is correct.

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