Application of the Kalman Filtering in Terrestrial Geodesy

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Key words: electronic tachymeter, Kalman filter, kinematic measurements, measurement noise, process noise

SUMMARY

One of the main tasks of geodetic surveying is measuring and analyzing deformations and movements of natural or man made objects. Nowadays geodetic deformation analysis means geodetic analysis of dynamic and kinematic processes (Welsch, Heunecke, 2001), which means incorporating the influencing factors and time dependence. Models describing the deformation as a function of time are kinematic models.

From the instruments point of view modern geodetic terrestrial instruments allow capturing dynamic (kinematic) process with high frequency of measurements. With proper deformation model the system state and its accuracy are definable in real time. Because the object is permanently in motion no redundant observations are available for the time point. The classical geodetic adjustment cannot be used. Instead, the filter methods have to be applied. The Kalman filter represents a method of advanced geodetic analysis of dynamic and kinematic processes. The estimation is based on predicting the future system state by using known past behaviour.

The objective of the article is to introduce the Kalman filter as an alternative method for estimation of kinematic geodetic measurements. On the basis of kinematic process simulation the model of linear Kalman filter is given in detail. The kinematic process is captured with motorized electronic tachymeter, which enables automatic tracking of reflector and measuring. In the numerical example the importance of initial filter parameters definition, with the emphasis on defining the process noise, is given.

From the numerical example, we can conclude that in the future work the emphasis should be given to research efficiency of the used instrument for kinematic measurements. Above all, a non-linear Kalman filter model, where the distance and angle measurement are directly processed, has to be developed. For the evaluation of the mathematical model the appropriate calibration system with known trajectory and velocity of the motion should be used. Such system would assure independent measure of certainty. It will also be possible to test the instrument's efficiency for kinematic measurements in dependence of measured values and velocity.

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1. INTRODUCTION

Geodetic surveying has since ancient times dealt with tasks that involve measurements related to objects in motion or objects seemingly in motion. In the past, the study of kinematic tasks was performed based on transformation of the kinematic problem into a sequence of static measurements. The results of subsequently processed static observations, performed at different points in time, are changes in the position of observed points of an object in the time between the chosen time points of observations. The advancement of kinematic surveying systems – Global Positioning System (GPS) and Terrestrial Positioning System (TPS) – introduced different instruments from different manufacturers, which enable automated, continuous monitoring of objects in motion and almost real-time positioning.

Conventionally, the procedures of studying deformation processes were in the past based on static and quasi-static models. The development of computer science, new surveying techniques and evaluation algorithms in the mid-1960s considerably changed the methods of deformation analysis. Besides the purely basic delineation of the geometric state, the methods included the temporal course of deformations and displacements; the so-called kinematic models were introduced. The kinematic approach to point coordinates as functions of time tries to describe the changes with coordinates, velocity and acceleration. In order to describe the movement of a point, one first needs to describe the trajectory as a function of time, as well as monitor the geodetic datum of the chosen coordinate system.

In the kinematic surveying technique we deal with temporal analysis and filtering techniques, which are based on adjustments of observations and give the relation between the observations and the unknowns in the stochastic and functional models. Since the position (x, y, z) in kinematic measurements is a function of time $r(t) = (x(t), y(t), z(t))$, the a posteriori processing based on a set of redundant observations is not possible, which necessitates the use of procedures that enable almost real-time evaluation of observations. The evaluation is based on predicting the a priori expected behaviour of a system, based on understanding of its past behaviour. In real time series the presence of noise is frequent, which disturbs the modeling and acquisition of optimal results. The standard techniques of noise removal from the time series include filtering and smoothing. In the sense of least squares estimation, the removal of noise from the time series has been optimized by the Kalman filter, which can be used only if we are familiar with the process equations (Lotrič, 2000).

In the continuation we represent the model of the linear Kalman filter and a simple case of its application for evaluation of kinematic measurements, performed by electronic tachymeter TCRA1105plus *Leica Geosystems*.

2. KALMAN FILTER

The theory of the Kalman filter was derived from the works of Gauss, Kolmogorov and findings of the Wiener filter in the beginning of the 1960s. In general, the purpose of filtering is to separate one object from another. The problem of filtering in engineering results from the electrotechnical field: determination of the signal in a specific frequency area and elimination of frequencies outside the defined area. Basically, the problem can be addressed in two ways:

- by choosing proper instruments, or
- by modeling an adequate mathematical algorithm for evaluation of desired and measured quantities.

In the 1940s Norbert Wiener addressed the problem from the mathematical aspect in trying to find the value of the filtered frequency that would enable an optimal separation of the signal from disturbance effects – noise. The result was the Wiener filter which was characterized as follows:

- assumption that signal and noise are random processes,
- criterion for best performance is minimum mean-square error,
- the solution is based on determination of optimal filter weighting function.

The result of the Wiener filter is the weighting function, which defines the weights of input values in such a way that the result is the optimal estimate of output quantities in a given moment. The algorithm provides an estimation of the signal for the previous moment – smoothing, estimation of the signal for the current moment – filtering, or prediction of the value of the signal for the next moment – prediction (Figure 1).

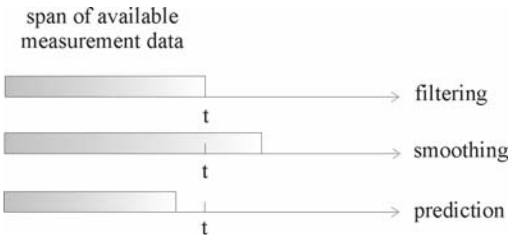

Figure 1: Three types of estimation problems; estimate desired at time t (Gelb, 1974)

The Wiener approach to filtering by the least-squares method is also used in the Kalman filter. Both models are based on the definition of the weighting function: how to weight the input data to ensure best estimate of the desired quantities in the present moment (Figure 2).

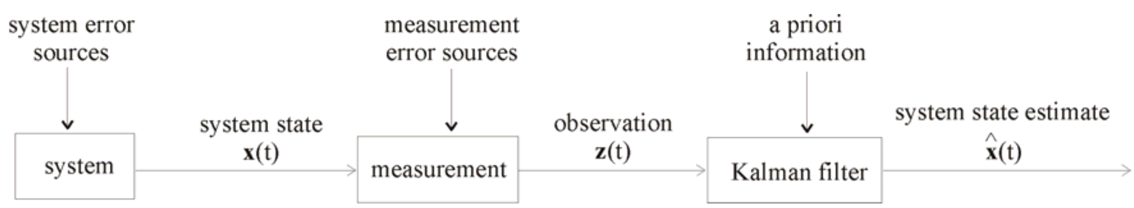

Figure 2: Block diagram depicting system, measurement and estimator (Gelb, 1974)

First practical applications of the Kalman filter were in the area of navigation and related to the definition of trajectories of a moving body in space and time.

The Kalman filter is an optimal recursive linear algorithm for processing the noisy measurements. For calculating the estimation of the system state and required values, the algorithm processes each observation no matter the accuracy. It includes all the available knowledge on:

- system dynamics and dynamics of surveying instruments,
- statistical characteristics of surveying errors, system errors and dynamics of the model, and
- initial values of wanted quantities.

In a recursive data processing the algorithm of the Kalman filtering does not require the storage of all measurements and states of the system, since in the calculation of the wanted quantities only the data from the previous step are required. This feature is the one basic advantage of the algorithm when applied to large systems with a large quantity of data. The model is linear. In case there are non-linear relations, they are linearized with development of functions into the Taylor series. The Kalman filter assumes that the noise of the system and measurements is white and Gaussian. The white noise is random process $\mathbf{e}_t$, where the variables are uncorrelated $E(\mathbf{e}_t \mathbf{e}_j) = 0, t \neq j$, with the zero-mean value $\mu_{\mathbf{e}}(t) = E(\mathbf{e}_t) = 0$ and variance $E(\mathbf{e}_t^2) = s^2$. If the noise is normally distributed, we refer to it as the Gaussian noise.

Let us observe the model of the Kalman filter. The Kalman filter deals with a random process, which is described by a linear process equation (Welch, Bishop, 2004):

$$\mathbf{x}_{k+1} = \mathbf{A} \cdot \mathbf{x}_k + \mathbf{w}_k \qquad (1)$$

and measurement equation:

$$\mathbf{z}_k = \mathbf{H} \cdot \mathbf{x}_k + \mathbf{v}_k \ , \qquad (2)$$

where:

$\mathbf{A}$ … state transition matrix	$\mathbf{x}_k$ ... system state vector at time t_k
$\mathbf{w}_k$ … process white noise	$\mathbf{H}$ … measurement matrix
$\mathbf{z}_k$ … vector of observations at time t_k	$\mathbf{v}_k$ ... measurement white noise.

Matrix $\mathbf{A}_{[n,n]}$ in process equation (1) connects the current value of the system state vector $\mathbf{x}_k$ at time t_k and the predicted value of $\mathbf{x}_{k+1}$ at time t_{k+1}. Matrix $\mathbf{H}_{[m,n]}$ in measurement equation (2) connects the system state vector $\mathbf{x}_k$ with observations $\mathbf{z}_k$. Matrices $\mathbf{A}$ and $\mathbf{H}$ can be functions of time, however they are assumed to be constant for most processes. The algorithm estimates the vector of wanted quantities $\mathbf{x}_k \in \Re^n$ (n is the number of unknowns for time step k) based on discrete observations $\mathbf{z}_k \in \Re^m$ (m is the number of measurements for time step k), at time t_k. Vectors $\mathbf{w}_k$ and $\mathbf{v}_k$ represent the system error and random measurement errors. For the variables it is assumed that they are white, and with normal probability distribution; $p(\mathbf{w}_k) \sim \mathrm{N}(0,\mathbf{Q}_k)$ and $p(\mathbf{v}_k) \sim \mathrm{N}(0,\mathbf{R}_k)$ accordingly. The corresponding covariance matrices $\mathbf{Q}_k$ and $\mathbf{R}_k$ of vectors $\mathbf{w}_k$ and $\mathbf{v}_k$ are given as:

$$R[\mathbf{w}_k \mathbf{w}_i^T] = \begin{cases} \mathbf{Q}_k, & i = k \\ 0, & i \neq k \end{cases}$$

$$R[\mathbf{v}_k \mathbf{v}_i^T] = \begin{cases} \mathbf{R}_k, & i = k \\ 0, & i \neq k \end{cases}$$

$$R[\mathbf{w}_k \mathbf{v}_i^T] = 0, \quad \text{for all } k \text{ and } i .$$

Through the process they can be constant or functions of time, changing with each measurement time t_k. The measurement covariance matrix $\mathbf{R}_{[m,m]}$ is based on the known accuracy of measurement systems or based on previous measurements. The determination of the covariance matrix of system $\mathbf{Q}_{[n,n]}$ is a more difficult one, since the process, which is being estimated, usually cannot be directly observed.

In deriving the Kalman filter the goal is to find equations that compute the a posteriori system state estimate $\hat{\mathbf{x}}_k$ as a linear combination of an a priori estimate $\hat{\mathbf{x}}_k^-$ and a weighted difference between the actual measurements $\mathbf{z}_k$ and the predicted measurements $\mathbf{H} \cdot \hat{\mathbf{x}}_k^-$:

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K} \cdot (\mathbf{z}_k - \mathbf{H} \cdot \hat{\mathbf{x}}_k^-) \quad . \tag{3}$$

The difference $\mathbf{d}_k = \mathbf{z}_k - \mathbf{H} \cdot \hat{\mathbf{x}}_k^-$ represents the innovation or the measurement correction and reflects the discrepancy between the predicted measurements $\mathbf{H} \cdot \hat{\mathbf{x}}_k^-$ and the actual measurements $\mathbf{z}_k$. The value of residual $\mathbf{d}_k = 0$ means a total agreement of the predicted and actual measurements. The question arises of how to define matrix $\mathbf{K}_{k\,[n,m]}$, the Kalman gain, which would enable the best – optimal a posteriori system state estimate. As a criterion of the estimate the minimum sum of squares of residuals was chosen, providing a solution of the optimal Kalman gain $\mathbf{K}_k$:

$$\mathbf{K}_k = \frac{\mathbf{P}_k^- \cdot \mathbf{H}_k^T}{\mathbf{H}_k \cdot \mathbf{P}_k^- \cdot \mathbf{H}_k^T + \mathbf{R}_k} \quad . \tag{4}$$

In case when the covariance matrix of observations $\mathbf{R}_k$ (diagonal elements yield the accuracy of observations, that is, the variances of observations) approaches zero, value $\mathbf{K}_k$ converges to $1/\mathbf{H}_k$. The actual observation $\mathbf{z}_k$ is trusted more and more, while the predicted value $\mathbf{H} \cdot \hat{\mathbf{x}}_k^-$ is trusted less and less. Reversely, as the a priori covariance of the system state vector $\mathbf{P}_k^-$ approaches 0, the vector of observations $\mathbf{z}_k$ is trusted less and less, and the predicted value of observation $\mathbf{H} \cdot \hat{\mathbf{x}}_k^-$ is trusted more and more. The entire Kalman loop is depicted in F.3. The Kalman filter iteratively corrects the Kalman gain $\mathbf{K}_k$ which makes the estimation of the state vector to converge towards an optimal solution. The estimation is optimal if $\mathbf{w}_k$ and $\mathbf{v}_k$ are zero-mean white noises with Gaussian distributions (Wira, Urban, 2000).
The Kalman filter estimates the process based on reverse control. The filter estimates the process unknown for a specific point in time, for which discrete observations with the presence of noise are performed. The filter is comprised of time equations of predictions and

observation equations – correction equations. The time equation gives an a priori system state estimate for the next time step, which is based on the current estimate of the unknowns and its covariance matrix. This is followed by a reverse control and correction of the a priori estimated values based on performed observations. The a posteriori system state vector estimate and its covariance matrices are calculated. Hence, the Kalman filter is an algorithm for solving numerical problems, based on predictions and updates of predictions.

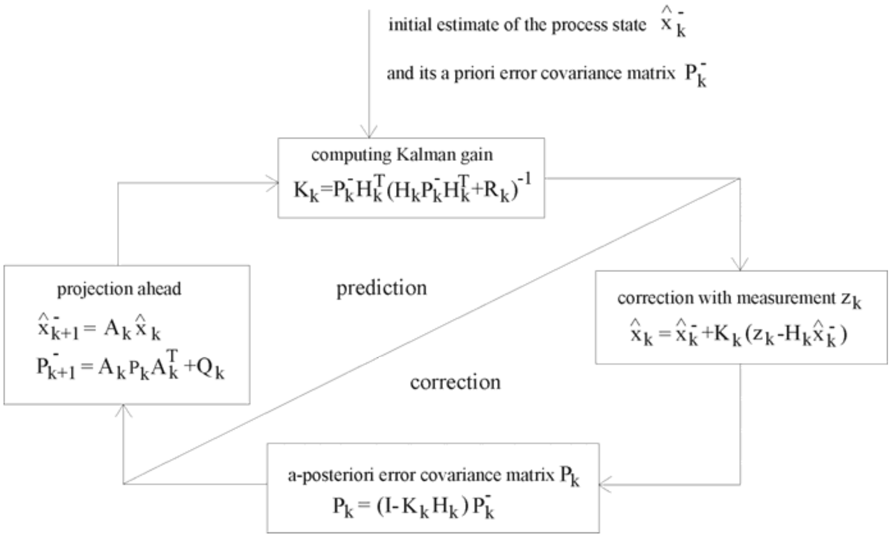

Figure 3: The Kalman loop (Brown, Hwang, 1992)

3. NUMERICAL EXAMPLE OF THE LINEAR KALMAN FILTER

3.1 Instrument and Setting-up of a Test Coordinate System

At the Faculty of Civil and Geodetic Engineering of the University of Ljubljana a test local coordinate system was set up, with a trajectory for simulation of reflector movement (F.4). The observations were performed with the electronic tachymeter TCRA1105plus (*Leica Geosystems*) in the LOCK mode, which enables the instrument to track the moving reflector. TCRA refers to the automatic tachymeter with automated tracking and pointing of the prism with an in-built IR and laser RL distance meter, which enables the reflectorless measuring of distances (http://www.geoservis.si/). By using the *AutoRecord* software (*Leica Geosystems*) that is installed in the instrument, the measurements are automated, with relation to the chosen criterion to be registered: time interval, change of distance, or time when no motion of the reflector is detected. The program was fitted especially for the instruments with the option of Automated Target Recognition (ATR) (Kogoj et al., 2004). For measurements the time criterion of registration was chosen (slope distance, horizontal angle, zenith distance), that is, the time interval $dt = 1\,\mathrm{s}$. The simulation of movement of 360°-reflector was performed manually. During the measurements, the instrument was mounted on pillar I with known spatial coordinates in the local coordinate system $(y, x, z) = (100{,}100{,}100)$. The initial direction – x direction – was defined with the direction towards pillar II. The y axis was directed towards pillar III. The axis of the movement of the reflector was approximately parallel to the

y axis. The observations were stored in the inner memory card of the instrument. The *.gsi file was transferred to the outer computer by using the *Leica SurveyOffice* software. The batch data of observations were then translated from the *.gsi text file of format 16 into *.txt text file, which is the input file in the Kalman filter algorithm.

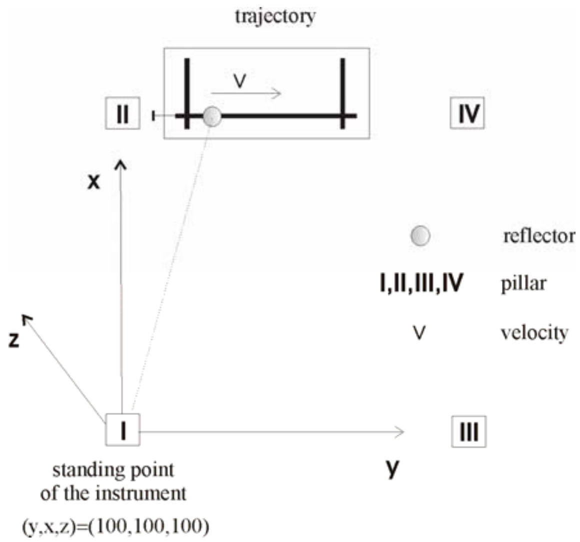

Figure 4: Local coordinate system

3.2 Model of the Linear Kalman Filter

In the text file the spatial coordinates are stored, as they are calculated by the instrument for each moment of measurements. In the linear Kalman filter the spatial coordinates represent the measurements $\mathbf{z} = \begin{bmatrix} x & y & z \end{bmatrix}^T$ and components of the system state vector – desired quantities $\mathbf{x} = \begin{bmatrix} x & v_x & y & v_y & z & v_z \end{bmatrix}^T$ (spatial coordinates of reflector state in a given moment and velocity in each direction) at the same time.

In the calculation of the Kalman filter unaccelerated motion is assumed, $a_x = a_y = a_z = 0 \text{ m/s}^2$. From the known equation of movement $s = s_0 + v * dt$ one can derive the process equation (1) as a matrix $\mathbf{x}_{k+1} = \mathbf{A} \cdot \mathbf{x}_k$:

$$\begin{bmatrix} x \\ v_x \\ y \\ v_y \\ z \\ v_z \end{bmatrix}_{[6,1]} = \begin{bmatrix} 1 & dt & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & dt & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & dt \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{[6,6]} \cdot \begin{bmatrix} x \\ v_x \\ y \\ v_y \\ z \\ v_z \end{bmatrix}_{[6,1]} . \qquad (5)$$

The equation of measurements $\mathbf{z}_k = \mathbf{H} \cdot \mathbf{x}_k$ is as follows:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{[3,1]} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}_{[3,6]} \cdot \begin{bmatrix} x \\ v_x \\ y \\ v_y \\ z \\ v_z \end{bmatrix}_{[6,1]} . \qquad (6)$$

For calculation of the Kalman filter algorithm with time equations of predictions and measurements equations of corrections the following values have to be defined:

- a priori system state $\mathbf{x}_{0\ [6,1]}$ and its covariance matrix $\mathbf{P}_{0\ [6,6]}$
- measurement covariance matrix $\mathbf{R}_{[3,3]}$, and
- process noise covariance matrix $\mathbf{Q}_{[6,6]}$.

The a priori system state of the spatial position of the reflector is based on previous measurements. The initial values of components of the velocity vector are determined as average velocities in single directions. For the determination of the a priori covariance matrix of the initial system state vector of unknowns $\mathbf{P}$ the accuracy of determination of initial position $0.01\,\mathrm{m}$ and velocity components $0.01\,\mathrm{m/s}$ are assumed. The covariance matrix of observations $\mathbf{R}$ is based on the capacity of the instrument in determining the position components captured with kinematic measurements; it is $\mathbf{R}_{[3,3]} = \mathrm{diag}(0.0001\,\mathrm{m}^2)$ and is independent of time. The process noise covariance matrix $\mathbf{Q}$ is the diagonal matrix $\mathbf{Q}_{[6,6]} = q \cdot \mathrm{diag}(1)$, where q is the appropriate scalar.

3.3 Interpretation of Results

The numerical case shows the dependence of results from the relation between the process noise (matrix $\mathbf{Q}$) and measurement noise (matrix $\mathbf{R}$). The accuracy of observations – matrix $\mathbf{R}$ – is constant through the process. The results are given for two different values of parameter q of the process noise covariance matrix $\mathbf{Q}$. The number of measurement epochs in both cases is $N = 48$.

The figures below depict the results for the value of $q = 0.00001$. F.5 represents the convergence of traces of matrix $\mathbf{P}$, whose diagonal elements are variances of computed system state values – components of position and velocity. The standard deviation of single position components converges to the value of $7.5\,\mathrm{mm}$ and velocity components to the value of $5\,\mathrm{mm/s}$ (F.6).

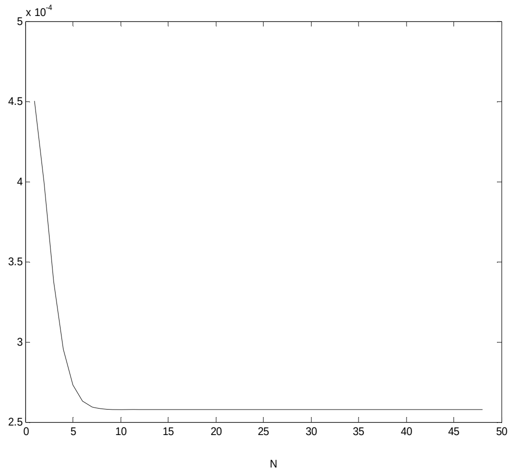

Figure 5: Trace of matrix $\mathbf{P}$ for $q = 0.00001$

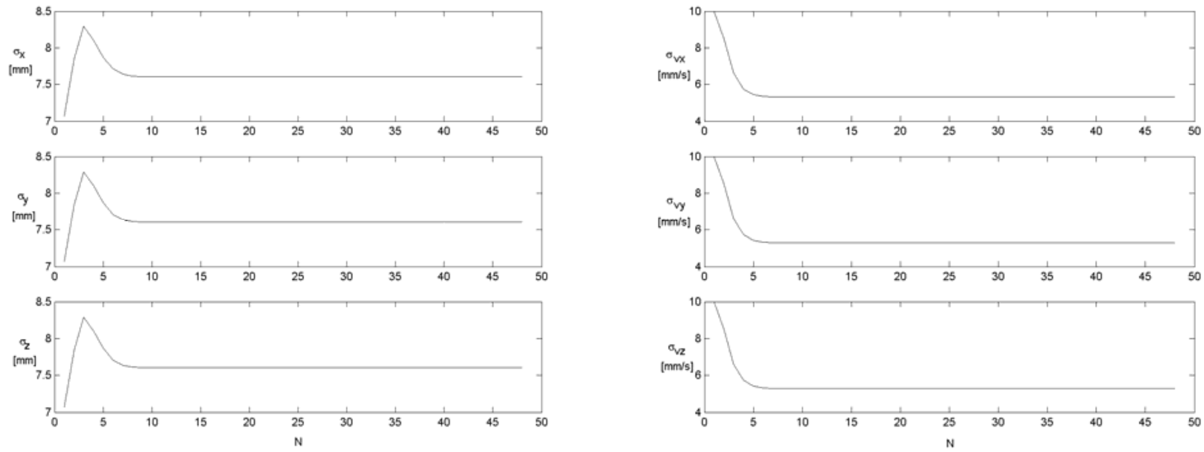

Figure 6: Standard deviations of position components and velocity components for $q = 0.00001$

F.7 represents the position components: measured, predicted and filtered values.

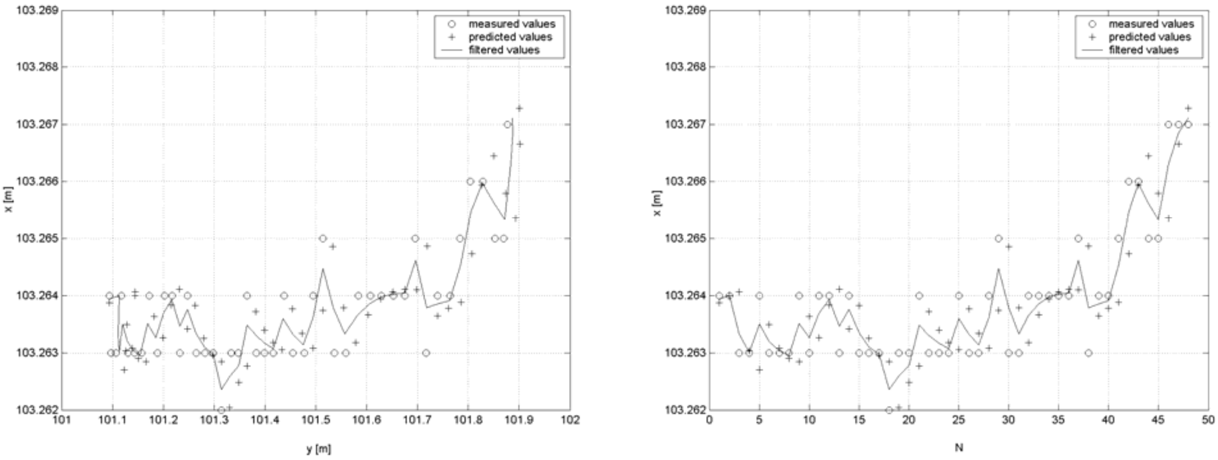

Figure 7: Measured, predicted and filtered values of position components of the reflector for $q = 0.00001$

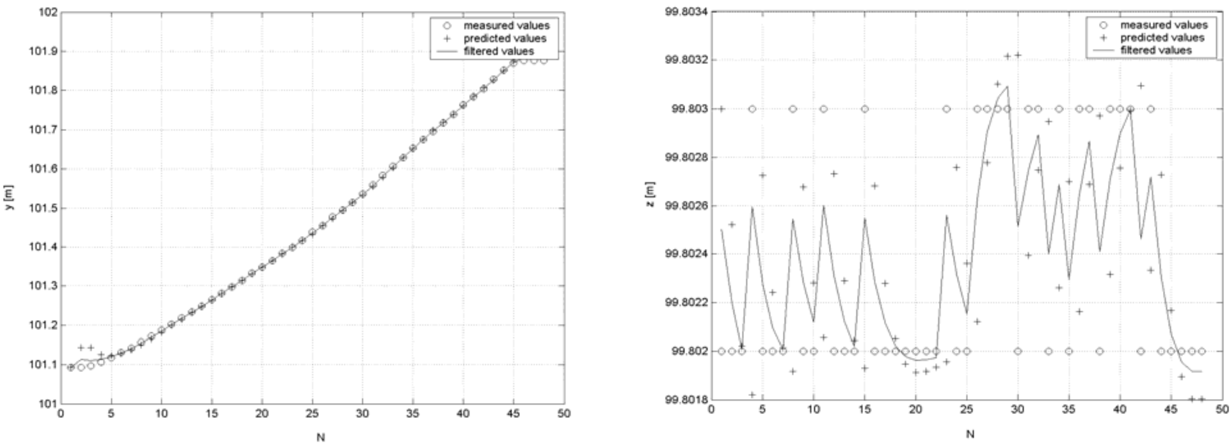

Figure 7: Measured, predicted and filtered values of position components of the reflector for $q = 0.00001$

From the velocity graphs (F.8) it is evident that the velocity in x direction and z direction was approximately zero, and in y direction approximately $v_y = 2\ \text{cm/s}$.

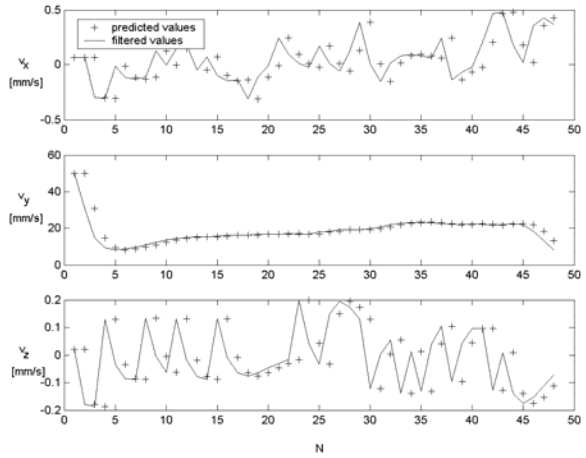

Figure 8: Velocity components for $q = 0.00001$

In the continuation a case of poor selection of process noise is presented, and a resulting filter divergence. The boundary value of $q = 0$ was chosen, indicating a full confidence into the mathematical model. Figures 9 and 10 show that the convergence of trace and standard deviations of components of spatial position and velocity were achieved. The standard deviation in positioning is three times better than in case $q = 0.00001$. Based on the results we could conclude that the model is fully fitted for evaluation of kinematic measurements. However, when looking at the y component (F.11), we can notice the occurance of filter divergence. The divergence occurs when the value of the covariance matrix $\mathbf{P}$, more specifically, of the diagonal elements, computed by the filtering process, become unacceptably small compared to the actual estimate of the unknowns (Sorenson, 1970). The discrepancy between the filtered and the measured values in y component (F.11) can be as much as $5\,\text{cm}$, which is, considering the capacity of the instrument and velocity of movements, considerably too high.

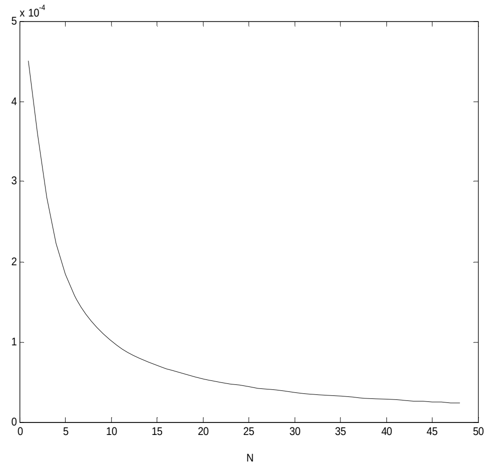

Figure 9: Trace of matrix $\mathbf{P}$ for $q = 0$

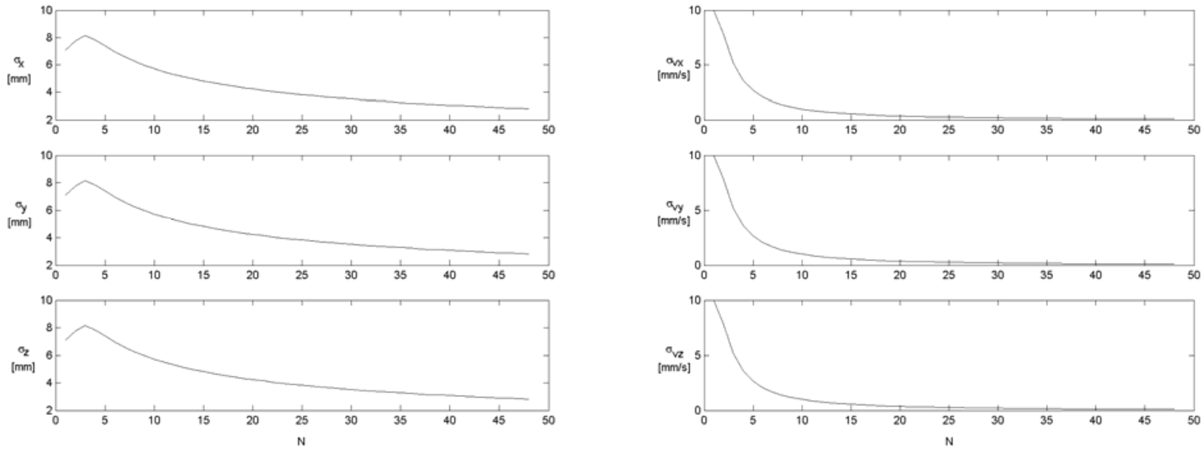

Figure 10: Standard deviation of position and velocity components for $q = 0$

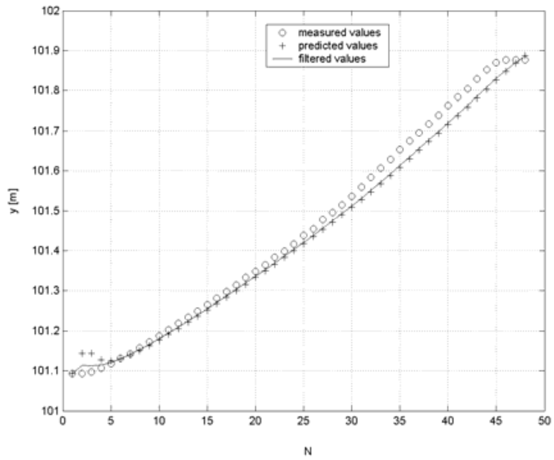

Figure 11: y component for $q = 0$

If we select the value of q too high, it may occur that the trace of matrix $\mathbf{P}$ converges to a high value (F.12) and as a result the standard deviations of unknowns are considerably too high as to the accuracy of measurements. The filtered values in such case follow the measured ones (F.13), since the level of confidence in observations in case of large q is higher.

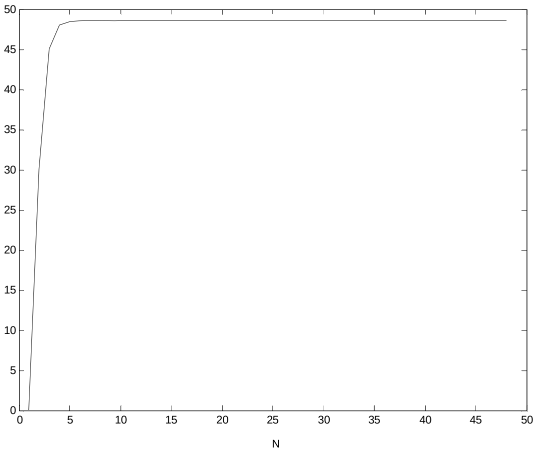

Figure 12: Trace of matrix $\mathbf{P}$ for $q = 10$

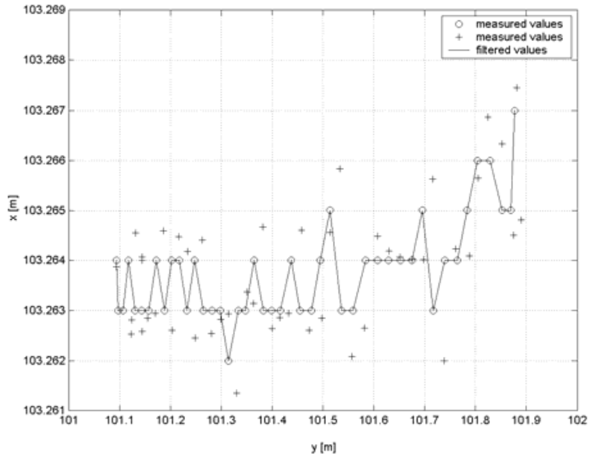

Figure 13: Measured, predicted and filtered values of reflector position at $q = 10$

4. CONCLUDING REMARKS AND PERSPECTIVES

In terrestrial surveying geodesy we have witnessed rapid development of geodetic instruments, resulting from higher capacity of measuring sensors and computer software. Besides the preliminary condition of knowledge on geodetic methods and understanding of procedures we can, by knowing the operation and capacity of measuring sensors and computer programs, considerably expand the possibilities of use of the terrestrial geodetic instruments. One of the possibilities is the capture of kinematic processes.

Due to the movement of the reflector, it is not possible to perform redundant measurements in kinematic measurements, which would reduce or remove the influences of the working environment, instrumental errors and negative effects of the mathematical model. In general, the requirements in kinematic measurements are more extensive ones. The choice of the instrument is key to the capture of kinematic (dynamic) processes. Also, one needs the select proper procedures for evaluation, which, beside the determination of unknowns, enable accuracy estimates. Hence, in kinematic measuring techniques we deal with time analyses and filtering techniques.

The efficiency of the mathematical algorithm of the Kalman filter has become widespread in geodesy, electrical engineering, medicine etc., that is, in tasks that require the estimation of unknown values as a function of time based on filtering of the noisy observations. In geodetic engineering, the efficiency of the mathematical algorithm is primarily in the tasks where we cannot perform redundant observations, that is, in the areas of navigation, continuous measurements of a moving object (monitoring of displacements and deformations).
Based on experience, we can conclude that in the kinematic measurements special attention should be given to:

- Capacity of the used instrument for kinematic measurements:

In the TPS1100 series of *Leica Geosystems* tachymeters the high dependence of positioning accuracy from the movement velocity of the reflector is identified. The deviation in the position can be as much as several centimeters (Stempfhuber, 2004), depending on the velocity, and is a result of the time delay during the measurement of angles and distances. In December 2005 an improved synchronization (TPS1200 Series), was introduced, where the time delay between the measurement of angles and distance is estimated at only several milliseconds.

- Adequacy of the model for evaluation – in this example Kalman filter model:

The convergence of traces and standard deviations is only the inner measure of reliability of the model. In future, one would need to set up a proper calibration system with a known trajectory, which would provide an estimation of the adequacy of the mathematical model and ensure the external measure of reliability. In such a system, one could test also the capacity of the instrument for kinematic measurements as a function of measured values and movement velocity.

- Non-linear filtering:

In future work some other techniques have to be researched. The linearized and extended Kalman filter model, where the observations of distances and angles are directly processed in filtering, has to be developed and compared with other nonlinear filters, such as unscented or particle filters.

- Adaptive techniques:

A well-known limitation of the Kalman filter is the assumption of known a priori statistics to describe the process and measurement noise, respectively. To overcome the fact that the model is not always available or that it can change online, the filter has to adapt itself so as to reflect the system dynamics without any a priori knowledge. One of the possible expansions of the Kalman filter, which can tackle the problem of imperfect a priori information, is adaptation through the filter learning process based on the innovation sequence (Mohamed and Schwarz, 1999).

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