

Recovery of the Ancient System of Foot/Cubit/Stadion – Length Units

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Key words:

SUMMARY

Statistical investigations of the length of nearly 870 collected metrological yard sticks have shown that their lengths can be assigned to about 30 length units; those length units can be derived back, using simple integer relations handed down from ancient times, to one basic unit, the so-called Nippur cubit = 518,5mm. Based on geometrical relations "ideal" lengths for all ancient cubit/feet-units will be derived and compared with mean values of the length of collected yard sticks. This system of ancient length units may be of interest to all concerned with the surveying of archaeological sites and buildings.

The definition of the ancient length unit "stadion" is 1 stadion = 600 feet. The foot units of yet existing archeological stadion sites will be identified. For example, the length of the stadion in Olympia (192,27m) is based on the so-called Remen of the old egyptian trade cubit OTC derived from the egyptian royal cubit (ORC = 523,75mm) : OTC = (24/28) ORC = 448,9mm, Remen OTC = (20/28) OTC = 320,7mm, "ideal" length of the stadion in Olympia = 600 Remen OTC = 192,4m.

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1. INTRODUCTION

For professional surveys of archaeological sites it is of utmost importance to recognize the fact that (nearly) all ancient buildings and towns are established according to an obviously normed set of precise non-metric ancient length units (cubit/foot or pechys/pous-units). For a reconstruction of this system of length units the information handed down by the ancient literature are as important as metrological yard sticks found at archaeological sites, e.g. in Nippur and Lagash in Mesopotamia.

Beside others the most important information in ancient literature are the tables added to the scripts of the Alexandrian scientist Heron from the 1. Century AD. (Heiberg 1972). In those tables six feet/pous units are listed, at least in one case two related to each other by the proportion 5:6. Separated into $(5/6) 16 = 13 \frac{1}{3}$ digiti/daktyloi were the “pous Italikos” (= $5/6$ pous Philetairikos) and the “pous Nikomedesios”, separated into 16 daktyloi the “basilikos”, pous Romaikos, pous Philetairikos and pous Ptolemaikos. The unit “pygon” (called “Remen” in Egypt) was defined by 1 pygon = 20 daktyloi. Very important for our understanding of the ancient non-metric set of length units are also the different definitions of a “pechys” (cubit)

| | |
|----------------------|----------------|
| pechys ephtymetrikos | = 24 daktyloi |
| pechys neilos | = 28 daktyloi |
| pechys histonikos | = 32 daktyloi |
| pechys thrakikos | = 34 daktyloi. |

Not mentioned in the metrological tables of Heron is the pygme = 18 daktyloi; maybe it was at his time already considered as an old and unimportant unit. Nevertheless, the relation $9:10 = 18:20 = 27:30$ or $3:4 = 18:24 = 24:32$ respectively seems to be also a very important one in the development of the sets of length units in ancient times.

Knowledge about the real length of the ancient units can of course provide only archaeological discoveries.

2. THE SETS OF ANCIENT LENGTH UNITS

In order to provide an overview the ancient length units are separated in the sequel into several groups only according to their mutual relations. To answer the question when and why the single units have been defined is a task of the history of exact sciences and technique.

2.1 The Old Egyptian Length Units

The development of those units is closely connected to the Egyptian method to mark off a square such as the ground-plan of the pyramid of Cheops; (Petrie 1934) found a description of this method in an old papyrus.

Based on the Remen ($R = (20/28) \text{ NC}$) a new length unit was defined, the Old royal cubit = $\sqrt{2} R = 523,75 \text{ mm}$.

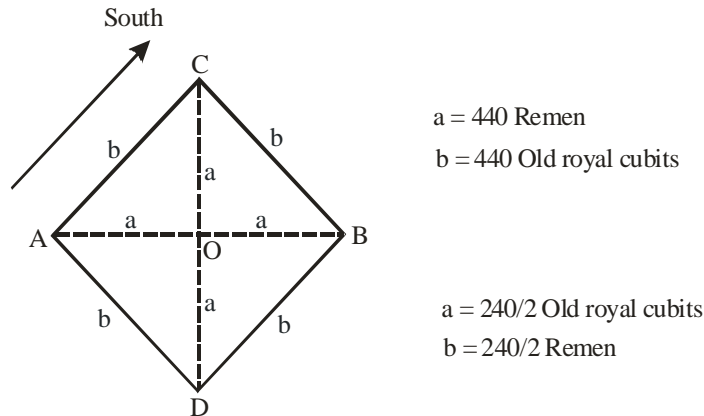


Fig. 1: Construction and mark off the pyramids of Cheops.

Starting from a point 0 at a straight line first the two points A and B have been marked by e.g. $a = 440 \text{ Remen}$. From A and B the two points C and D have been fixed using $b = 440 \text{ old royal cubits}$. With two controls

- are $OC = OD = 440 \text{ Remen}$ and
 - form the points COD a straight line
- it could be checked that ABCD form really a square.

Without doubt astro-geodetic techniques have been applied to build the pyramid of Cheops. The ground sides are aligned in meridian direction with an accuracy of a few minutes of arc. Fixing the height to be $h = 280 \text{ old royal cubits}$, the zenith distance of the edges of the pyramid are

$$\tan^{-1}\left(440/280\sqrt{2}\right) = 48^{\circ}0' = 2\varepsilon,$$

where ε is the inclination of the ecliptic at about 2500 b.c. This explains the length of 440 R.

The small pyramids (of the wife of Cheops), accompanying the big one, have a base length b of 44,45 m, that is nearly $240/2 R = 44,43 \text{ m}$; the diagonals are therefore $240/2 \text{ old royal cubits}$. Indeed, the so-called trade cubit was $TC = (24/28) RC = 448,9 \text{ mm}$.

The pyramids of Cheops are an illustrative example, how important the knowledge of the ancient length units is for archaeological investigations.

| name | definition | length (mm) |
|-----------------------|--------------|-------------|
| Remen R | (20/28) NE | 370,35 |
| Old royal cubit ORC | $\sqrt{2}$ R | 523,75 |
| Old trade cubit OTC | (24/28) ORC | 448,9 |
| Remen trade cubit RTC | (20/28) OTR | 320,7 |

Tab. 1: Old egyptian length units

2.2 The Megalithic Yard System

Other examples are the megalithic sites in Europe such as Stonehenge, as investigated in particular by the British engineer A. Thom and his son A.S. Thom. Characteristic seems to be the subdivision of the Nippur cubit as well as the Remen into 30 parts.

$$\begin{aligned} \text{Megalithic Yard MY} &= 2 (24/30) \text{ NE} = 2 \cdot 414,8 \text{ mm} = 829,6 \text{ mm} \\ \text{Megalithic Remen} &= 2 (24/30) \text{ R} = 2 \cdot 296,3 \text{ mm} = 592,6 \text{ mm} \end{aligned}$$

as well as

$$\begin{aligned} \text{Salamis cubit} &= (28/30) \text{ NE} = 483,9 \text{ mm} \\ \text{attic-olympic foot} &= (18/30) \text{ NE} = 311,1 \text{ mm}. \end{aligned}$$

As shown in table 2, several of the greek/roman length units are connected to the megalithic Remen by simple relations

| name | definition | length (mm) |
|-----------------------------|----------------------|-------------|
| Megalithic Remen MR | (24/30) R | 592,6 |
| pes romanus (Pous Romaikos) | ($\frac{1}{2}$) MR | 296,3 |
| Cubitus romanus CR | ($\frac{3}{4}$) MR | 444,4 |
| Dodrans | ($\frac{1}{2}$) CR | 222,2 |
| Pes Drusianus | ($\frac{3}{4}$) CR | 333,3 |
| Pechys metrios | (24/30) MR | 474,1 |
| Pous metrios | (16/30) MR | 316,1 |
| Pechys basilikos | (27/30) MR | 533,3 |
| Pous basilikos | (18/30) MR | 355,5 |

Table 2: Megalithic/Roman/Greek length units

Recognise the conservative and long tradition of this non-metric system from the relation
 English foot = (24/28) Pous basilikos = 304,75 mm.

2.3 The Gudea-System

A key for our understanding of the length units of the hellenistic/roman epoch is without doubt the question about the length of two foot units, frequently mentioned in the ancient literature:

Pous Italikos = $(5/6)$ Pous Philetairikos.

Both have been probably fairly wide spread and known to everybody. Under the units handed down from ancient times there is indeed one which was widely distributed and was most probably the

Pous Philetairikos = $(6/5)$ GU = 317,5 mm

This length unit was known in Rome as “Palmipes romanus”, it is known from Milet (foot of Miletus) and also in China as “Tschì”, closely connected to the Royal Cubit by

Pous Philetairikos = $(18/30)$ RC = $(18/30)$ 529,1 mm = 317,5 mm

A pechys corresponding by the relation 2:3 would be

“Pechys Philetairikos” = $(27/30)$ RC = 476,2 mm.

Of course, the Gudea unit was called then Pous Italikos by the Greek/Romans.

The length units connected by simple proportions to the Gudea unit are listed in table 3.

| name | definition | length (mm) |
|----------------------------------------------|-------------------|-------------|
| Gudea unit / Pous Italikon GU | $(20/28)$ R | 264,55 |
| Pous of Kyrenaika | $(20/24)$ R | 308,6 |
| Royal cubit RC | 2 GU | 529,1 |
| Pous Ptolemaikos | $(20/30)$ RC | 352,7 |
| Pous Philetairikos (Tschì, palmipes romanus) | $(18/30)$ RC | 317,5 |
| Pechys Philetairikos? | $(27/30)$ RC | 476,2 |
| Shaku/Japan (Stadion in Milet) | $(16/28)$ RC | 302,3 |
| Pous of new Carthago | $(5/6)(20/30)$ RC | 293,9 |

Table 3: Length units connected to the Gudea unit (Pous Italikon)

2.4 The Babylonian-Ionian System

The system is characterised by the relation

Babylonian cubit = $(30/32)$ Royal cubit = 496,0 mm

or

Ionian foot = $(30/32)$ Remen = 347,2 mm

Several length units are connected with this ones by simple relations as shown in table 4.

| name | definition | length (mm) |
|-----------------------------------|-----------------|-------------|
| Babylonian cubit BC | (30/32) RC | 496,0 |
| Babylonian trade cubit BTC | (27/30) RC | 446,4 |
| Heraion foot (Hebraic foot) | (2/3) BTC | 297,6 |
| Foot of the stadion in Priene | (20/28) BTC | 318,9 |
| Foot of the Zeustemple in Olympia | (2/3) BC | 330,7 |
| Oskian-umbrian foot | (5/6)(2/3) BC | 275,6 |
| Foot of the stadion in Milet | (5/6)(20/28) BC | 295,2 |
| Ionian foot | (30/32) R | 347,2 |
| Pergamon cubit? | (3/2)(30/32) R | 520,8 |

Table 4: Babylonian-Ionian length units

One question remains open: What was the “Pous Nikodemesios” in the table of Heron? Because it should be in any case, as well as the pous Italikon, a small unit, a possible candidate would be the foot unit found in South-Italy, in Oskian and Umbrian, but without more information this can only be a presumption.

3. ON THE PROVENANCE OF THE LENGTH OF THE STADION IN OLYMPIA

Several archaeological stadion sites have survived the turbulent times from the greek/roman epoch until now; the length of their racetracks are listed in table 5.

| stadion | length (m) | foot | length (mm) | ideal length | rel. dev. |
|-----------|------------|---------------------|-------------|--------------|-----------|
| Olympia | 192,27 | Remen eg. tr. cubit | 320,7 | 192,42 | 0,8 ‰ |
| Epidauros | 181,30 | “Shaku” | 302,3 | 181,38 | 0,4 ‰ |
| Priene | 191,39 | --- | 318,9 | 191,34 | 0,3 ‰ |
| Milet | 177,36 | --- | 295,2 | 177,12 | 1,4 ‰ |
| Delphi | 177,55 | Pes romanus | 296,3 | 177,78 | 1,3 ‰ |
| Athens | 184,96 | Pous of Kyrenaika | 308,6 | 185,16 | 1,1 ‰ |

Table 5: Race track length of stadion sites

Having recovered the system of cubit/foot units, the foot unit for building the race tracks could easily be identified.

In the year of the Olympic games in Athens the length of the stadium of Olympia is of course of main interest. As a greek myth tells us it was build according to the length of the foot of Herakles himself.

There remains another question: Was it the very old egyptian god Herakles or the younger greek hero Herakles? As is well-known from the tales of Herodot, the egyptians worshipped a god named Herakles long before the greek Herakles lived.

The simple figures leave no doubt: The stadion in Olympia was designed and build according to the foot length of the egyptian Herakles.

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