



## INDIRECT DETERMINATION OF THE INVARIANT REFERENCE POINT (IVP) OF SLR AND VLBI OBSERVING SYSTEMS

Alexander R. WOODS

*Geoscience Australia, Australia*

**Abstract:** The integrity and strength of multi-technique terrestrial reference frames such as ITRF2005 depend on the precisely measured and expressed local tie connections between space geodetic observing systems at co-located observatories. Australia has several geodetic observatories which together host the full variety of space geodetic techniques, including GPS, GLONASS, DORIS, SLR and VLBI. The observational reference point of large geodetic observing systems, such as SLR and VLBI, are generally inaccessible. Therefore, Geoscience Australia developed an indirect measurement approach, using terrestrial observations, to precisely determine the invariant reference point (IVP) of SLR and VLBI telescopes. The indirect IVP determination technique involves a rigorous process of three-dimensional circle-fitting to the coordinates of targets observed on the structure during rotational sequences about each of the systems' independent axes (e.g. azimuth and elevation). Geoscience Australia routinely measures millimetre accurate connections between survey monuments and geodetic observing systems at co-located observatories across Australia. The IVP derivation technique continues to be refined, looking to account for further un-modelled systematic errors.

### 1. INTRODUCTION

The combination of space geodetic observing systems at co-located observatories is fundamental to the realisation of the International Terrestrial Reference Frame (ITRF) (Sarti, et al., 2004). Precisely measured and expressed local terrestrial connections between complementary space geodetic observing systems, such as GPS, GLONASS, DORIS, SLR and VLBI, provide the link between co-located techniques. The connection is made through a survey commonly referred to as a local tie survey. Highly accurate local ties have become more and more important as the achievable accuracies of the space geodetic techniques have improved. As well as supporting multi-technique terrestrial reference frame development, local tie connections, at a precision approaching one millimetre, also enable determination of relatively independent estimates of measurement accuracy and the detection and modelling of technique dependent biases (Long and Bosworth, 2000).

Damage to telescope structures or any man-made changes, such as critical maintenance, can lead to permanent displacement of the telescope reference point, which causes discontinuities in the geodetic time series (Nothnagel et al., 1997). Accordingly, Haas and colleagues (2000) discuss the importance of continually undertaking local tie surveys to precisely determine the displacement of VLBI reference points caused by repair work to telescope foundations and mechanical parts. These sudden displacements, and more subtle displacements brought about



over extended periods, should be monitored with respect to the surrounding survey network as precisely as possible through detailed local tie surveys.

However, in the case of SLR and VLBI observing systems, it is particularly difficult to define and thus measure a system's measurement reference point, which is generally inaccessible and cannot be directly observed. Stakeholder organisations have thus developed various direct and indirect survey techniques in attempts to obtain high precision connections to large telescope structures at geodetic observatories.

For example, Geoscience Australia has developed an indirect measurement approach, using terrestrial observations to precisely determine the invariant reference point (IVP) of SLR and VLBI telescopes. In this paper, the IVP of SLR and VLBI telescopes is defined as the intersection of the azimuth axis with the common perpendicular of the azimuth and elevation axes (see also Johnston et al., 2004) and is a position which does not change when the telescope is rotated (Sarti et al., 2004). The indirect IVP determination technique involves a rigorous process of three-dimensional circle-fitting to the coordinates of targets observed on the structure during rotational sequences about each of the systems' independent axes (e.g. azimuth and elevation). Geoscience Australia routinely measures millimetre accurate connections between survey monuments and geodetic observing systems at co-located observatories across Australia. These surveys also support critical site deformation monitoring and assessment. The Australian indirect IVP derivation technique continues to be refined, in attempts to account for further un-modelled systematic errors such as thermal deformation and gravitational structural deformation.

## 2. LOCAL TIE TECHNIQUES

Various different local tie techniques have been investigated at co-located sites around the world (Sarti et al., 2004). Local tie surveys are most commonly undertaken using a combination of conventional terrestrial survey techniques (e.g., total station, precise levelling) and space geodetic techniques (e.g., GPS). Organisations have developed different techniques that range from attempts to directly observe telescope reference points by placing a target at the approximate physical location of the reference point, to more rigorous indirect observation techniques that also support determination of axis alignment and offsets as well as an estimate of the telescope reference point. The selection of survey techniques depends on the site, survey equipment and survey expertise available, as well as the awareness of surveyors of the complexity and necessity of accurate telescope IVP estimation.

In general, applied local tie survey techniques involve the analysis of results of conventional geodetic adjustment of one or several targets observed on the telescope during rotational sequences. For example, the local tie antenna reference point determination procedure adopted by NASA surveyors for VLBA antennae involved taking observations from a network of ground control stations to a target positioned at the approximate vertical axis of rotation, and simultaneous observations to a target rod mounted on the apex of the antenna quadripod, as the antenna was rotated in elevation for four separate azimuth settings (Long and Bosworth, 2000). A circle-fitting procedure was applied to the separate target arcs and the mean of the circle centres was used to determine the elevation axis offset. However, this technique relies primarily on the correct placement of targets at specific locations on the



antenna structure, which may not be accessible on other antennas and misplacement of targets may lead to poor repeatability of survey results.

An indirect reference point measurement approach has been developed for the 100m Effelsberg (Germany) radio telescope, where observations are made to one eccentric spherical target mark located adjacent to the inaccessible telescope reference point (Nothnagel et al., 1999). Simultaneous observations are taken from two-three total station standpoints to the target mark as the telescope is rotated in azimuth and elevation. The data processing stage involves adjusting target coordinate measurements for telescope thermal expansion and gravitational deformation before modelling the behaviour of the eccentric target mark with antenna rotation and determining the invariant reference point through least squares analysis. The Japanese GSI adopted a similar local tie technique for the connection between VLBI and GPS reference points at geodetic observatories in Japan (Matsuzaka et al., 2004). A precise local tie was obtained from conventional terrestrial survey observations made from a network of ground stations to the centre of a spherical shell on which a target on the telescope traces as the antenna is rotated through various orientations (Matsuzaka et al.).

Sarti and colleagues (2004) detail a particularly rigorous and comprehensive method of estimating the eccentricity vectors and associated variance-covariance matrices between observing systems at co-located observatories, focusing on reference point determination of SLR and VLBI telescopes (with or without intersecting axes). The method used at Medicina (Italy) involved observing several targets on the telescope as it was rotated through various axial rotations and then applying geometrical (rotational motion of the telescope) constraints and symmetry considerations in a three-dimensional least squares analytical procedure to derive the telescope reference point. The horizontal coordinates and height of the telescope reference point were computed from planimetric coordinates of the horizontal circle centres and height of the vertical circle centres. The placement of targets on the telescope and processing considerations aimed to account for subtle tilt and axes offsets in the telescope, which may bias IVP estimates. The Medicina local tie surveys were well planned, with dual observations taken from two total stations set up on a network of permanent observation pillars to targets on the telescope structure. The derived eccentricities for Medicina were transformed from the local reference frame to ITRF and output in a SINEX file format for inclusion as an additional set of observations in ITRF multi-technique adjustment (Sarti et al.).

In parallel to the work of Sarti and colleagues (2004), Geoscience Australia has developed a similar technique which is routinely applied at the Australian co-located geodetic observatories, and is detailed in the following section. Comparison of adjustment and processing techniques for data from Yarragadee (Australia) and Medicina (Italy) indicated overall good agreement of 0.5mm between the two IVP models with minor differences noted (Dawson et al., 2007).

### 3. AUSTRALIAN TECHNIQUE

Geoscience Australia is responsible for maintaining the local tie connections at the co-located geodetic observatories throughout Australia. Terrestrial connections to an accuracy of 1mm (in the local frame) are routinely observed and include the rigorous determination of the SLR or VLBI telescope invariant reference point (IVP). This process involves the derivation of the independent axes of rotation of the telescope through a process of three-dimensional circle-

fitting to the three-dimensional coordinates of targets observed on the telescope during rotational sequences.

At each co-located site observations are made from a precisely determined ground network, which is routinely monitored as part of the local tie survey to ensure a consistent, stable terrestrial network from which accurate and over-determined local tie connections can be made between survey monuments and geodetic observing systems. Repeating local tie surveys every two to three years allows an estimate of true survey accuracy to be quantified, as well as supporting the monitoring of site deformation between survey epochs.

There are various limitations and impediments which can complicate reference point determination. Often these are site specific, but examples include:

- many telescopes have limited rotational freedom, reducing the reliability of axis determination;
- the fixed (or primary axis) and the moving (or secondary axis) axis may not intersect (and are in some cases many metres apart);
- the system's structural design may constrain visibility, limiting line of sight observations to targets and influencing network design; and
- telescopes are often in high demand with full observation schedules. Observatory managers need to understand the importance of the local tie connections and allow sufficient amounts of time and support for surveys to be completed successfully.

A general description of the observation process involves a series of measurements to targets positioned on structural components of the telescope under study as it is moved through several rotational sequences. Generally, the structure is rotated at incremental steps (10-20 degrees, see Figure 1) through its full range of motion in azimuth orientations while being held fixed in zenith, and through the full range of zenith orientations while being held fixed in an azimuth setting.

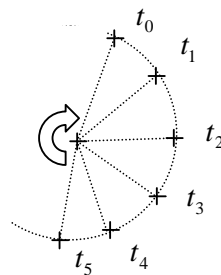


Figure 1 - Incremental target observations of a rotating telescope.

This procedure is followed in order to monitor targets as they scribe a perfectly circular arc, as a target located on a rigid body rotating about one independent axis can be used to express a circle in 3D space. Observations to the targets are used to derive the parameters of three-dimensional circles in space, which can be described by seven parameters (Figure 2), namely:

- Circle centre (three parameters,  $x_m, y_m, z_m$ )
- Unit normal vector (three parameters,  $n_x, n_y, n_z$ ) perpendicular to the circle plane
- Circle radius (one parameter,  $r$ )

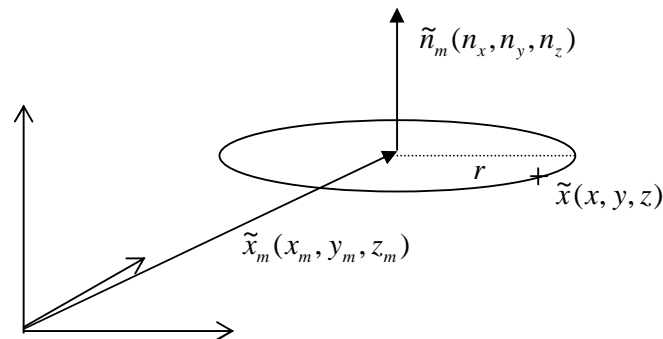


Figure 2 - The seven parameters of a circle in space.

It is essentially from the intersection of these circles that the IVP is determined (see Figure 3).

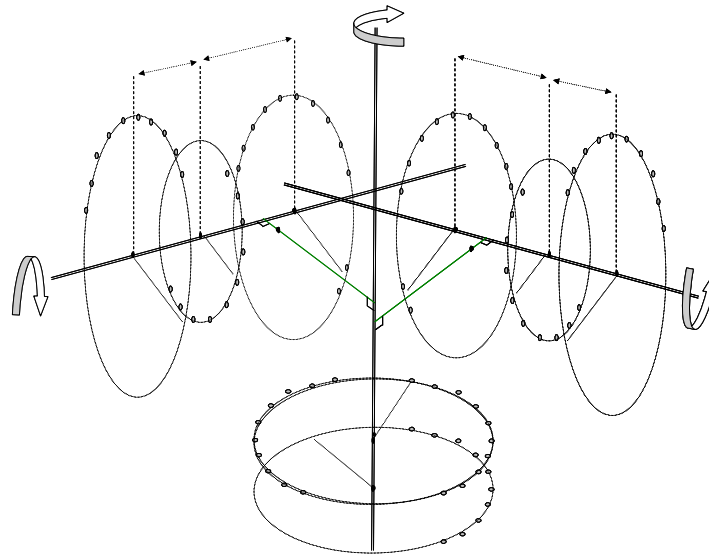


Figure 3 - IVP model.

In the adjustment process, geometrical models describing target motion during rotational sequences about each of the systems' independent axes (azimuth and elevation) are applied and include:

- target paths scribe a perfect circle in 3D space during rotation about an independent axis;
- normal vectors to each circle plane derived from targets rotated about the same axis are forced to be parallel (Figure 4);
- circle centres derived from targets rotated about the same axis are forced to lie along the same line in space (Figure 5);
- the unit normal vector perpendicular to the plane of the circle is assigned magnitude one;
- orthogonality (or non-orthogonality) of the elevation axis to azimuth axis remains constant over all realisations of the elevation axis;
- identical targets rotated about a specific realisation of an axis will scribe three-dimensional circles of equal radius (Figure 5);

- offset distance between the elevation axis and azimuth axis remains constant over all realisations of the elevation axis;
- distance between three-dimensional circle centres for all realisations of the elevation axis are constant over all realisations of the elevation axis; and
- IVP coordinate estimates remain constant over all realisations (combinations) of the azimuth/elevation axis.

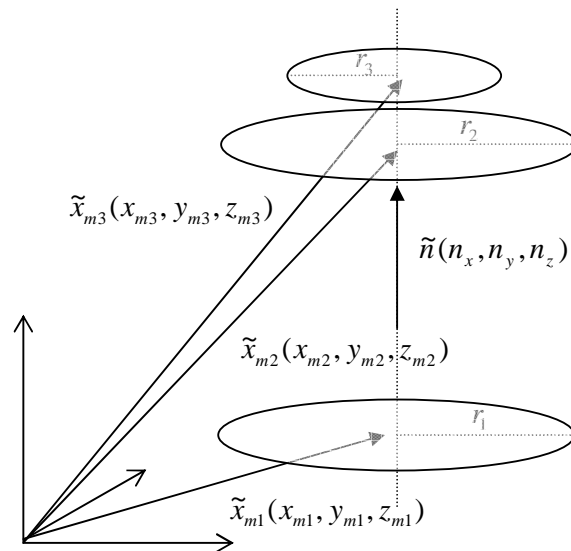


Figure 4 - Multiple circles along an axis, share common normal vector parameters.

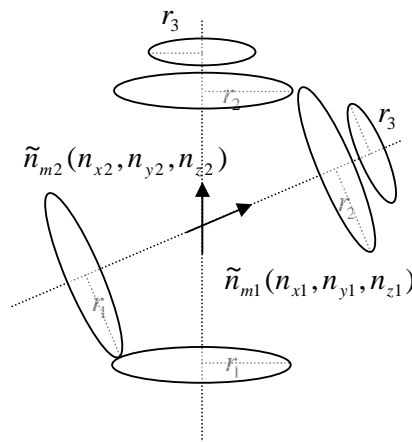


Figure 5 - Multiple circles along multiple axes, where the target circle radius remains consistent for each orientation.

These conditions are critical to the computation of unbiased IVP coordinates at the millimetre level for a telescope where the rotational freedom is limited by visibility and/or structural constraints (Dawson et al., 2007).

The method of IVP determination assumes that during rotational sequences targets follow a perfect circular arc in 3D space, there is no deformation of targeted structure during rotational

sequence, there is no axis wobble error, and the axis of interest can be rotated independently of the other axis (thus one axis can be held fixed while the other moves through a full rotational sequence). There are no assumptions of target symmetry about the system reference point, axis orthogonality, verticality, horizontality or the precise intersection of the axes made using this IVP estimation technique.

Software developed at Geoscience Australia is used to undertake a rigorous least squares analysis that utilises all target coordinates, their variance-covariance information and the constraints listed above for the derivation of circle parameters and the computation of the axes of rotation and their intersection to determine the final system IVP. During the IVP modelling and estimation process, additional system parameters can also be derived. Estimates can be obtained for the azimuth axis deflection from the vertical, the orthogonality (or non-orthogonality) of the azimuth to the elevation axes and the offset distance between the azimuth and elevation axis. The IVP modelling procedure also provides an assessment of the circle fit residuals, listing a measure of the in-plane and out-of-plane residuals for each target at each telescope orientation (see Figure 6). Residual analysis and outlier detection can highlight any slight change in telescope orientation during the survey. All these parameters are of particular interest to the telescope operators and technicians.

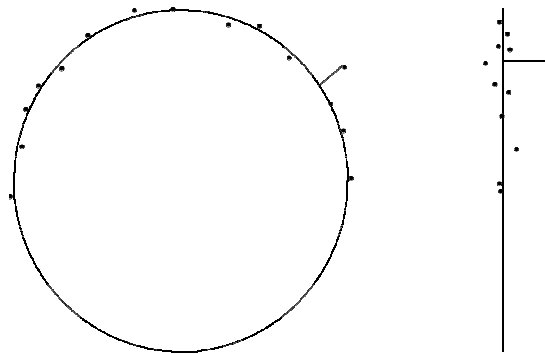


Figure 6 - Circle fitting residuals; left in-plane residuals; right out-of-plane residuals.

The analysis software has the capability to transform (through translation and rotation only) the terrestrial network and computed IVP coordinate with the variance-covariance matrix from a local to a global reference frame, including a geocentric variance-covariance matrix. Three alignment stations with XYZ earth-centred Cartesian coordinates (derived from GPS processing) are specified as control stations for the local to global reference frame transformation. The final adjustment results, including the millimetre level accurate connections and their associated variance covariance matrix, can then be output in a SINEX format solution file, suitable for submission to the International Earth Rotation Service (IERS).



Comparisons between the 2003 and 2007 Yarragadee Moblas 5 local tie survey results (Woods and Ruddick, 2007) show average residuals of 0.8mm, 0.7mm and 0.6mm in the East, North and Up components. In particular, the position of the determined SLR IVP varied between years by 0.7mm, 1.1mm and 0.3mm in the East, North and Up directions. The strong agreement in the IVP position between epochs illustrates the high accuracy achievable with the indirect IVP determination technique.

#### 4. FORWARD IMPROVEMENTS

Although the documented local tie survey technique for SLR and VLBI telescope reference points provides consistent high precision connections between observing systems, a number of potential advancements in survey techniques and data processing have been identified. These include consideration of methods for aligning surveys to global reference frames without loss of accuracy and for taking the different external forces that can deform a telescope structure into account. The most critical of these influences are thermal deformation and structural deformation induced by gravitational sag. In addition, Sarti and colleagues (2004) propose further work to improve local tie connections, such as studying the structural deformation of the sub-reflectors of VLBI telescopes with rotation and verifying the stability of local ground control networks.

In relation to methods for aligning surveys to global reference frames, Dawson and colleagues (2004a) note that the indirect local tie survey technique supports terrestrial tie survey precision of approximately one millimetre (in the local frame). However, further error due to the alignment to the ITRF is more difficult to quantify but is assumed to be of the level of three millimetres and is dependent on the quality/correctness of the associated GPS analysis used for alignment to the global frame. Matsuzaka and colleagues (2004) also identify the alignment of the local tie vector (orientation and vertical direction) to the global reference frame using short (day long) GPS campaigns as the major source of error in the local tie procedure adopted by the Japanese GSI. This discussion recommends multiple day observation campaigns and comprehensive GPS analysis for highly reliable GPS vectors for the transformation process.

Research was conducted into the gravitationally-induced structural deformation of the Hobart (Australia) radio telescope with changes in elevation orientation and the need to model these effects, as a significant source of inter-technique bias, to improve local tie connections (Dawson et al, 2004b). The study highlighted the need to model for the effect of gravitational sag, which was shown to vary the distance between the radio telescope's reference point and receiver by as much as ~2mm. Gravitational sag has potential to change the electrical path through the radio telescope system, impacting geodetic VLBI parameter estimation. This study flagged the need for research and development of models of structural deformation of large telescopes that are applicable to terrestrial survey connections.

In addition, Geoscience Australia is investigating structural thermal expansion modelling procedures in the IVP determination process. An approximate bias of 2mm in the vertical at the VLBI telescope IVP is estimated for un-modelled thermal expansion effects (Dawson 2005). Given the high accuracy requirements of local tie surveys, VLBI antenna thermal deformation corrections need also be applied to the reduction of terrestrial survey data.





Specifically thermal corrections need to be applied to each individual target observed throughout the period of the terrestrial survey.

However, correction of terrestrial survey data for antenna thermal deformation is somewhat different from the conventional model approach employed in VLBI data analysis (e.g., *IERS 2003 Conventions*: McCarthy and Petit, 2004), as the thermal correction is dependent upon which structural elements each target is attached to and the correction needs to be expressed in three dimensions. Adopting the principles of the conventional VLBI data analysis thermal modelling, Geoscience Australia has developed a thermal deformation model for application to survey data. In this model, the antenna is divided into four separate structural elements, including the antenna foundation, antenna pillar, rigid structure about the primary axis and rigid structure about the secondary axis. The antenna thermal model accounts for various parameters, which include temperature dependent coefficients of expansion for each of the structural elements, the instrument's reference temperature, the air temperature profile before and during the terrestrial survey, target positions (determined during classical geodetic adjustment), and several telescope axis parameters (Dawson, 2005). Invariant point determination is not expected to be improved significantly by the application of thermal corrections (i.e. as measured by individual/independent survey analysis) except when large temperature variations occur during survey operations. However, survey-to-survey invariant point coordinate repeatability and accuracy is expected to improve as a result of the application of structural thermal corrections.

## 5. CONCLUSION

High precision local tie surveys are fundamental to the realisation of global reference frames. Various survey techniques have been developed to obtain precise terrestrial survey connections between observing systems, with particular attention paid to the difficulty of IVP determination for SLR and VLBI telescopes. Survey techniques vary in their complexity although the most commonly regarded method involves an adaptation of indirect determination through a procedure of three-dimensional circle fitting and intersection, followed by rigorous least squares adjustment and the application of geometrical constraints. The field survey procedures and IVP determination techniques have been refined, although continuing research needs to be directed towards accounting for unmodelled external forces. Geodetic observatory stakeholders need to be aware of the importance of the precise local tie surveys and ensure adequate connections are established and maintained. Investigation into more precise and efficient survey techniques and modelling procedures continues.

## References

- Dawson, J., Johnston, G. and Twilley, B. (2004a). *The determination of telescope and antenna invariant point (IVP)*, Unpublished report, Geoscience Australia, Australia, p. 1-8.
- Dawson, J., Titov, O., Johnston, G., Naebkhil, S. and Woods, A. (2004b). *The impact of gravitational sag on VLBI results of the Hobart Mt. Pleasant radio observatory*, Unpublished report, Geoscience Australia, Australia, p. 1-30.
- Dawson, J. (2005). *VLBI antenna thermal deformation modelling for terrestrial survey data*, Unpublished report, Geoscience Australia, Australia, p.1-5.



Dawson, J., Sarti, P., Johnston, G. and Vittuari, L. (2007). Indirect approach to invariant point determination for SLR and VLBI systems: as assessment. *Journal of Geodesy*, 81, p. 433-441.

Haas, R., Nothnagel, A. and Behrend, D. (2000). VLBI determinations of local telescope displacements, *IVS2000 General Meeting Proceedings*, p. 133-137.

Johnston, G., Dawson, J. and Naebkhil, S. (2004). *The Mount Stromlo local tie survey*. Record 2004/20, Geoscience Australia, Australia, p. 1-26. At URL:  
[http://www.ga.gov.au/image\\_cache/GA5653.pdf](http://www.ga.gov.au/image_cache/GA5653.pdf) (viewed 13/03/08).

Long, J. L. and Bosworth, J. M. (2000). The importance of local surveys for tying techniques together, *IVS2000 General Meeting Proceedings*, p. 113-117.

Matsuzaka, S., Masaki, Y., Tsuli, H., Takashima, K., Tsutsumi, T., Ishimoto, Y., Machida, M., Wada, H. and Kurihara, S. (2004). VLBI-GPS co-location results in Japan, *IVS2004 General Meeting Proceedings*, p. 138-142.

McCarthy, D. and Petit, G. (eds.) (2004). VLBI antenna thermal deformation. In: *IERS Technical Note No.32, IERS Conventions (2003)*, Frankfurt am Main: Verlag des Bundesamts für Kartographie und Geodäsie, p. 86-88.

Nothnagel, A., Wirtz, C., Sauerbier, M. and Sobanski, M. (1999). Local survey at the Effelsberg radio telescope 1997 – Preliminary results, *13<sup>th</sup> Working Meeting on European VLBI for Geodesy and Astrometry*, p. 25-31.

Sarti, P., Sillard, P. and Vittuari, L. (2004). Surveying co-located space-geodetic instruments for ITRF computation. *Journal of Geodesy*, 78, p. 210-222.

Woods, A. and Ruddick, R. (2007). The 2007 Yarragadee (Moblas 5) local tie survey. Record 2007/19, Geoscience Australia, Australia, p. 1-27. At URL:  
[http://www.ga.gov.au/image\\_cache/GA10608.pdf](http://www.ga.gov.au/image_cache/GA10608.pdf) (viewed 13/03/08).

#### **Corresponding author contacts**

Alexander Raymond WOODS  
alex.woods@ga.gov.au  
Geoscience Australia  
Australia