DATUM DEFINITION AND ITS INFLUENCE

ON THE SENSITIVITY OF GEODETIC MONITORING NETWORKS

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Abstract: The sensitivity of a geodetic network is defined as its capacity to detect and measure movements and deformations in the area covered by the network. This paper attempts to study the effects of datum definition on the sensitivity of geodetic networks. Particular attention is paid to the geometry of the datum points and to its effect on the sensitivity of the network. Principles from continuum mechanics are used to analyze the datum definitions required for a sensitive network, and it is shown how the sensitivity of a monitoring network is strongly influenced by the geometry of the network points. This paper presents a new perspective and describes relevant parameters that enable defining and quantifying the influence of the datum on the sensitivity of geodetic networks. Following an introduction of the concept of geodetic networks' sensitivity and a development of theoretical tools, the paper presents results of numerical experiments carried using a schematic horizontal GPS network. These results indicate that the sensitivity of a geodetic network depends on the geometrical distribution of the network points and on the chosen set of points that define the datum of the network.

1. Introduction

The primary goal of deformation analysis according to our approach is the determination of velocities of points located on a deformed area. The velocities are unique parameters capable of representing the deformation phenomenon [10]. The velocities model the reality of deformation as a four dimensional phenomenon.

Estimation of the velocities parameters can be done directly from the geodetic measurements or indirectly by the two-steps analysis approach [3] [9]. In both methods the raw measurements are used to estimate the coordinates of the network points, for a reference epoch t_0 and the velocity of the points, $\dot{\mathbf{x}}$.

The observation equations for solving $\mathbf{x}_0$ and $\dot{\mathbf{x}}$ are rank deficient due to the lack of datum. There is a need to define a reference coordinate systems for $\mathbf{x}_0$ and $\dot{\mathbf{x}}$, and thus the datum defect is double. The datum defect is corrected by adding a corresponding number of constraints to the unknown parameters. Since the position axis of the network points is perpendicular to the time axis, there is no dependence between the datum definition of the coordinates of the network points ($\mathbf{x}_0$) and the velocity of the points ($\dot{\mathbf{x}}$). Consequently, changes in the datum definition, when defining $\mathbf{x}_0$, do not affect the solution of $\dot{\mathbf{x}}$. The constraints that apply to the network points could be different from those applied to the velocities, hence it is not necessary for the two reference systems to coincide. As we are interested mainly in the velocities, our concern remains with the selection and definition of a meaningful reference system for $\dot{\mathbf{x}}$.

A unique solution of the datum problem can be achieved by adding a minimum number of constraints that equals the number of the datum defect. The defect of a 3D velocities network is three when the definition of origin is missing and it can grow to seven when the definition of the rotations and scale are missing as well. Geodetic measurements contain part of the datum definition, therefore the exact size of the network defect is dependent on the type of the existing measurements. GPS measurements can be assumed to have an orientation and scale, therefore the size of the datum defect in a GPS network is three, since the definition of origin is missing. For each monitoring campaign we may use the datum parameters that are contained in the GPS measurements for estimating the coordinates of the network points, but not for estimating the velocities in the deformation analysis. Velocities are estimated based on a time series of monitoring campaigns. Fluctuations in the GPS orbits could affect the orientation and scale between monitoring campaigns [2]. Therefore, when calculating velocities we should not obtain the datum definition from the GPS measurements but rather assume a datum defect of seven parameters, $d = 7$.

2. Datum Definition and S-transformation

According to Wolf [11] we can transform one solution, $\hat{\mathbf{x}}$, pertaining to a certain datum into another , $\underline{\hat{\mathbf{x}}}$, pertaining to another datum using a similarity transformation. Let I be the identity matrix and **G** a similarity transformation matrix, also known as Helmert's transformation matrix. Such a transformation, also known as the S-transformation, is described by:

$$\underline{\hat{\mathbf{x}}} = [\mathbf{I} + \mathbf{G}\mathbf{E}]\hat{\mathbf{x}} = \mathbf{J}\hat{\mathbf{x}} \tag{1}$$

while $\mathbf{E} = -(\mathbf{G}^{\mathrm{T}}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}$. $\underline{\hat{\mathbf{x}}}$ is the unique solution that yields $\underline{\hat{\mathbf{x}}}^{\mathrm{T}}\underline{\hat{\mathbf{x}}} \rightarrow \min$. Note that $\mathbf{J}$ is idempotent: $\mathbf{J} = \mathbf{J}^2$ and $\mathbf{J} = \mathbf{J}^{\mathrm{T}}$.

The cofactor matrix for the solution $\hat{\mathbf{x}}$ is called **Q**. In accordance with the law of error propagation, the cofactor matrix $\underline{\mathbf{Q}}$ of the transformed solution $\underline{\hat{\mathbf{x}}}$ is

$$\underline{\mathbf{Q}} = \mathbf{J}\mathbf{Q}\mathbf{J}^{\mathrm{T}}. \tag{2}$$

The solution $\underline{\hat{\mathbf{x}}}$ and its cofactor matrix $\underline{\mathbf{Q}}$ are based on a datum defined by all the points in the network. It is the optimal datum as the trace of the cofactor matrix $\underline{\mathbf{Q}}$ is minimal [6]. However, the application of a datum definition that relies on all the points in the network is not sensible.

Let $\mathbf{P_x}$ be a diagonal matrix with 1 for points that enter the datum definition and 0 for all others. When searching for a solution with $\underline{\hat{\mathbf{x}}}^{\mathbf{T}}\mathbf{P_x}\underline{\hat{\mathbf{x}}} \rightarrow \min$ we get:

$$\mathbf{J} = \mathbf{I} - \mathbf{G}(\mathbf{G}^{\mathrm{T}}\mathbf{P}_{\mathrm{x}}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{P}_{\mathrm{x}}. \tag{3}$$

The projector **J** is no longer a symmetric matrix since $\mathbf{G}(\mathbf{G}^{\mathrm{T}}\mathbf{P}_{\mathrm{x}}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{P}_{\mathrm{x}}$ is not symmetric.

A conventional datum for the velocities $\dot{\mathbf{x}}$ is determined by identifying a subset of network points that are congruent, thus the relative positions of those points are invariant in time. The datum is defined by minimizing the following quadratic form:

$$\dot{\mathbf{x}}^T P_X \dot{\mathbf{x}} \rightarrow \min . \quad (4)$$

The S-transformation is a very convenient tool for defining a subset of congruent network points. When dealing with geodetic networks the Helmert transformation matrix **G** usually contains columns as the number of the network datum defect. In 3-D networks the first three columns pertain to the translations, the additional three columns define the rotations and the last column defines the scale of the network, totaling seven columns. In the case of spatial deformation monitoring networks, when translations, rotations, scales and obliquities have to be defined in the area covered by the network, we should extend the **G** matrix due to the additional need to define the scale factor for the three axes and the angles between them, a total of twelve columns, $d = 12$ [8]. The following section shows how the extended Helmert transformation matrix has the same form as the matrix used to decompose the velocity field into the rigid motion and homogenous strain parameters.

3. The Sensitivity Criteria

The sensitivity of a network is defined as its capacity to detect and measure movements and deformations in the area covered by the network.

We examine the sensitivity of the network using the statistical test of hypothesis. The null hypothesis

$$H_0 : \quad \dot{\mathbf{x}} = 0$$

can be tested against the alternative hypothesis

$$H_1 : \quad \dot{\mathbf{x}} \neq 0 .$$

If the null hypothesis is accepted there is no movement of points in the network. The cofactor matrix $\mathbf{Q}_{\hat{\dot{x}}}$ has rank r and f degrees of freedom in the estimation of $\hat{\dot{\mathbf{x}}}$ and the a posteriori variance factor is $\hat{\sigma}_0^2$. The test statistic including all velocities of the network points is:

$$t = \frac{\hat{\dot{\mathbf{x}}}^T \mathbf{Q}_{\hat{\dot{x}}}^+ \hat{\dot{\mathbf{x}}}}{r \hat{\sigma}_0^{\,2}} \sim F_{r,f} . \quad (5)$$

For three-dimensional networks with u points $r = 3u - d$, where d can equal 7 or 12, depending on the Helmert transformation used (regular or extended), and for two-dimensional networks $r = 2u - d$, where d can equal 4 or 6.

If $t > F_{r,f,\alpha}$ the null hypothesis is rejected, where α is the level of significance. This means that at least one point has moved significantly. The sensitivity of the network is increased as t increases. The test statistic t can increase when the velocities ($\hat{\dot{\mathbf{x}}}$) increase or when the velocities accuracies increase (when the standard deviations are smaller).

The design of a monitoring network based on the sensitivity criteria has been presented by Even-Tzur [5].

4. The Sensitivity of a Group of Points

Principles from continuum mechanics can be used to analyze the datum definition required for a sensitive network [1] [7]. In a three-dimensional analysis of the point velocity field we compute the three parameters of the velocity of the network's barycenter in the reference system $(\bar{\dot{x}} \quad \bar{\dot{y}} \quad \bar{\dot{z}})$, the three rotation parameters $(r_x \quad r_y \quad r_z)$ and the deformation rate tensor. The deformation tensor is composed of the scale factors of the three axes $(d^{-1}_{xx} \quad d^{-1}_{yy} \quad d^{-1}_{zz})$ and the angles between them $(d_{yz} \quad d_{xz} \quad d_{xy})$, for a total of 12 parameters:

$$\mathrm{g}^T = \Big[\underbrace{\bar{\dot{x}} \quad \bar{\dot{y}} \quad \bar{\dot{z}}}_{\textit{translation}} \;\Big|\; \underbrace{r_x \quad r_y \quad r_z}_{\textit{rotation}} \;\Big|\; \underbrace{d^{-1}_{xx} \quad d^{-1}_{yy} \quad d^{-1}_{zz}}_{\textit{scale}} \;\Big|\; \underbrace{d_{yz} \quad d_{xz} \quad d_{xy}}_{\textit{obliquity}} \Big]. \tag{6}$$

We introduce a matrix **B** with the following composition:

$$\mathbf{B}^T = \begin{bmatrix} 1 & 0 & 0 & 0 & z_1 & -y_1 & x_1 & 0 & 0 & 0 & z_1 & y_1 \\ 0 & 1 & 0 & -z_1 & 0 & x_1 & 0 & y_1 & 0 & z_1 & 0 & x_1 \\ 0 & 0 & 1 & y_1 & -x_1 & 0 & 0 & 0 & z_1 & y_1 & x_1 & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 1 & y_u & -x_u & 0 & 0 & 0 & z_u & y_u & x_u & 0 \end{bmatrix}. \tag{7}$$

The coordinates $\mathbf{x}_i$, $\mathbf{y}_i$ and $\mathbf{z}_i$ of point i $(i = 1, 2, ...u)$ are given in a Cartesian system that is parallel to the reference system and with an origin at the network's barycenter. It should be noted that the matrix **B** has the same form as the matrix **G** (the extended Helmert matrix).

The velocity field of a congruent group of points can be partitioned into a linear model $\mathbf{B}^T\mathbf{g}$ and a residual vector **v**:

$$\dot{\mathbf{x}} = \mathbf{B}^T\mathbf{g} + \mathbf{v}\,. \tag{8}$$

The velocity vector $\dot{\mathbf{x}}$ has a covariance matrix $\mathbf{Q}_{\dot{x}}$. According to Papo [7] the vector **g** and its covariance matrix $\mathbf{Q}_g$ are given as

$$\begin{aligned} \mathbf{g} &= (\mathbf{B}\mathbf{Q}_{\dot{x}}^{-1}\mathbf{B}^T)^{-1}\mathbf{B}\mathbf{Q}_{\dot{x}}^{-1}\dot{\mathbf{x}} \\ \mathbf{Q}_{\mathbf{g}} &= (\mathbf{B}\mathbf{Q}_{\dot{\mathbf{x}}}^{-1}\mathbf{B}^T)^{-1}. \end{aligned} \tag{9}$$

For a 3-D network at least four non-coplanar points are required in a group and for a 2-D network three non-collinear points are required to define all the elements of vector **g**.

In a four-dimensional network, the property that distinguishes the datum points from the rest of the network is the fact that their relative velocities are smaller than a certain significance level. The matrix $\mathbf{Q}_g$ can serve as a tool to examine the sensitivity of a group of points representing a block in the network.

By using the linear part of the model for the velocity field (as presented in equation 8) we can define the test statistic (5) for a congruent group of points as

$$t = \frac{\hat{\mathbf{g}}^T(\mathbf{B}\mathbf{Q}_{\hat{\dot{x}}}^{-1}\mathbf{B}^T)\hat{\mathbf{g}}}{r\hat{\sigma}_0^{\,2}} \sim F_{r,f}\,. \tag{10}$$

For a group of points $\hat{\mathbf{g}}$ can be regarded as invariant, r is a constant, and $\hat{\sigma}_0^2$ is the variance factor. Hence the product $\mathbf{B}\mathbf{Q}_{\dot{x}}^{-1}\mathbf{B}^T$ defines the sensitivity of the group of points. The larger the trace of $\mathbf{B}\mathbf{Q}_{\dot{x}}^{-1}\mathbf{B}^T$ is, the more sensitive the network is. In other words, the more accurate **g** is the more sensitive the network is.

The accuracy of **g** depends on the accuracy of the velocity field, which for GPS networks is independent of the geometric distribution of the network points. However, the matrix **B** is obviously dependent on the network geometry.

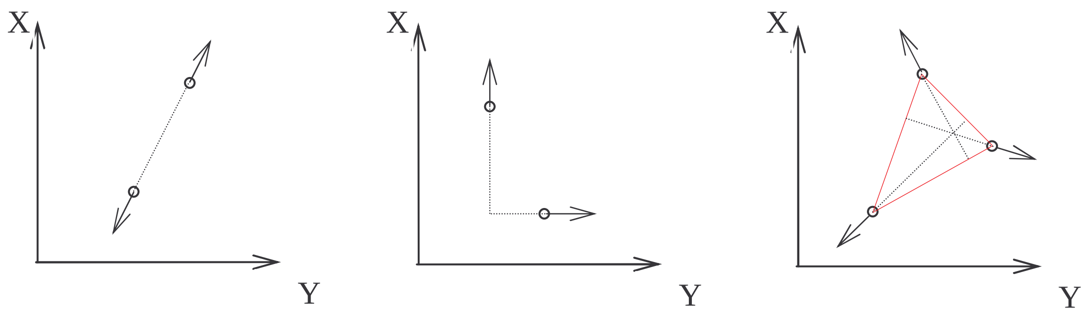
In order to determine the best ability for monitoring individual components of vector **g** we shall examine the optimal geometrical distribution of points according to our sensitivity criteria, using a two-dimensional network for the sake of simplicity. The interested reader can find that for three-dimensional networks the inferences are almost similar. The inferences presented below were obtained from experiments carried and based on an examination of the characteristics of matrix **B**.

Translation - A single point can define the linear velocity of a certain area covered by the network. So the ability to determine the velocity of the barycenter of one of the network blocks is not influenced by the geometrical distribution of points.

Rotation - Two points are needed to determine the rotation of a block. The more distant these points are, the better the rotation can be detected, as seen in Figure 1 (a).

Scale change - Two points, A and B for example, with $X_A = X_B$ or $Y_A = Y_B$, can not distinguish a difference in the X or Y scale. We aim for a coordinate difference between points that will be as large as possible in both components, as seen in Figure 1 (b).

Variation of angle between axes - Three points are needed to detect the angle between the two axes in a two-dimensional network. The optimal distribution of points is when the distance in both coordinate components is as large as possible, as seen in Figure 1 (c).



(a) Rotation (b) Scale (c) Angle between axes

Figure 1: The best ability for monitoring individual components of vector g.

Matrix **B** must be a full row rank, otherwise vector **g** is undefined. Generally there are more points than needed for a unique evaluation of each component of **g**. This allows us to examine the results and to evaluate their error estimation.

It should be noted that in a three-dimensional network the difference in the vertical components is always small, around a few hundred meters. Therefore we can expect low sensitivity in this direction.

There are three types of measurements that provide a reasonable level of accuracy for monitoring deformation, GPS vectors, distances and height differences. For these types of measurements the estimation of the measurement variances depends on the distance between the points, as the variance increases when the distance increases. Therefore, the wish for a long distance between points for increasing the network sensitivity should be limited. The conclusion as to the best distribution is based on the assumption that the covariance matrix remains the same when the distances increase. However, our analysis should give us an idea of what we should aspire for.

5. Datum Definition and its Influence on Sensitivity

Let us investigate the influence of datum definition on the sensitivity of the network, and explore the relationship between the datum points' geometry and the sensitivity of the network points.

Let two groups of points be located in two different blocks separated by a fault line. Measurements are taken between the points. For simplicity, we assume that the geometrical distribution of each group is identical relative to the fault line. The points represent the blocks and the measurements allow determining the relative motion and deformation between the blocks. Let us also assume that each group of points can serve as a datum, assuming that the relative position of each group of points is invariant in time. Let us investigate which group of points should define the datum when the objective is achieving a sensitive network. Arbitrarily we can define one group of points as a reference block (datum). The second block contains the object points (referred to as the object block). Movements and deformation of the object block are defined in relation to the reference block. In this situation the sensitivity of the network can be increased if the geometrical distribution of the object points will be wide, as described in the previous section.

If the object points can serve as datum points, we can transform their velocities, using the S-transformation, and use them as the reference points. If translation is considered between the reference block and the object block, matrix **J** (see equation 3) is independent of the geometrical distribution of the datum points as matrix **G** is independent of the points' position. Therefore, the sensitivity of the network is not dependent on the reference block chosen, and each block can serve as a datum where the network sensitivity is not influenced by the geometrical distribution of points.

If rotation is considered, the situation is more complex. Let us assume that the object block rotates around the barycenter of the object points relative to the reference block. The rotation can be expressed as velocities according to equation (8). We use the S-transformation between the object block and the reference block. Now the velocities of the object points that were used as reference points are used to evaluate the vector of rigid motion and homogenous strain parameters (**g**) of the object block, using equation (9). Now the new object block (previously referred to as the reference block) rotates in the opposite direction to the original rotation but at the same rate, and has a translation as well. Due to the S-transformation the rotation of the original reference block is done around the barycenter of the original object points. Consequently, the velocities are greater and the test statistic t (equation 5) increases. Therefore, as long as the barycenter of the reference block is close to the rotation axis, the sensitivity of the object points (as their barycenter is distant from the rotation axis) increases. In this situation the sensitivity of the network increases as the distance between the object points increases.

When the object block is deformed due to a scale change or obliquity of the axes and we use the S-transformation and turn it into the reference block, the new object block becomes deformed in an opposite rate and also has a translation. The velocities of the new object block are greater and the test statistic t increases. As long as points on the deformed block are used to define the datum, the sensitivity of the object points increases. In this situation the sensitivity of the network increases when the coordinate difference between points is as large as possible in both components, and the distance in both coordinate components is as large as possible.

The conclusions regarding the influence of datum definition on network sensitivity are based on the assumption that the covariance matrix of the velocities remains the same when datum transformation is implemented between blocks. In homogenous monitoring deformation network this assumption is generally valid.

Thus, it becomes clear that the geometric distribution of the datum points does not dramatically affect the sensitivity of the network. The geometric distribution of the object points should be further considered, but, as strange as it may seem, it is important to establish the datum based on points that represent the deformed block.

The datum should provide a solid basis for defining movements of points. Wide geometrical distribution of the datum points is better than a narrow geometrical distribution in order to achieve a better accuracy of the points in the network [8].

6. An Example

A small two-dimensional schematic GPS network containing 8 points for monitoring crustal deformation is shown in Figure 2. Let us assume that due to tectonic activity block II moves relative to block I with a rate of 2 millimeters a year in the north direction and rotates around its barycenter with a rate of 1×10^{-6} radian a year. Let block I be the reference block and block II the object block. The postulated vector g, based on the tectonic activity at the monitoring region, is:

$$\mathbf{g}^{\mathrm{T}} = \begin{bmatrix} 2_{\mathrm{mm/yr}} & 0 & 1\times10^{-6}{}_{\mathrm{rad/yr}} & 0 & 0 & 0 \end{bmatrix}$$

The linear part of the velocity field of the points on block II can be computed according to equation (8). Then the 16 by 1 velocity vector of all network points is:

$$\dot{\mathbf{x}}^{\mathrm{T}} = [\underbrace{0 \quad 0 \quad \ldots \quad 0}_{\text{block I}} \,\vdots\, \underbrace{7 \quad 10 \quad 7 \quad -10 \quad -3 \quad 10 \quad -3 \quad -10}_{\text{Block II}}]_{\mathrm{mm/yr}}$$

All vectors were measured between the network points. The variance (in meters) of a vector of length ℓ_{ij} meters between points i and j is given by $\sigma_{\ell_i} = 0.003 + \ell_{ij} \times 0.5\mathrm{ppm}$, and a correlation of 10% is assumed between the two vector components. Zero correlation is assumed between any two different vectors. The weight matrix was produced using a variance of a unit weight equaling 1, $\sigma_0^2 = 1$. Let the time interval between two epochs of observations be one year, $\Delta t = 1_{\mathrm{yr}}$. The cofactor matrix of the velocity vector $\dot{\mathbf{x}}$ is

$$\mathbf{Q}_{\dot{x}} = \frac{\mathbf{Q}_{x_1} + \mathbf{Q}_{x_2}}{(\Delta t)^2}. \qquad (11)$$

When points located on block I define the datum, the numerator of equation (5) is equal to $\dot{\mathbf{x}}^{\mathrm{T}}\mathbf{Q}_{\dot{x}}^{+}\dot{\mathbf{x}} = 45.6$.

The S-transformation is implemented on the velocities and their cofactor matrix to change the reference block into block II. The velocity vector of all network points is now:

$$\dot{\mathbf{x}}^{\mathrm{T}} = [\underbrace{-27 \quad -10 \quad -27 \quad 10 \quad -17 \quad -10 \quad -17 \quad 10}_{\text{block I}} \,\vdots\, \underbrace{0 \quad 0 \quad \ldots \quad 0}_{\text{Block II}}]_{\mathrm{mm/yr}}.$$

A decomposition of the velocity field using equation (9) into the rigid motion and homogenous strain parameters shows that block I moves with a rate of -22 millimeters a year in the north direction relative to block II, and rotates with a rate of -1×10^{-6} radian a year. The numerator of equation (5) when block II defines the datum is equal to $\dot{\mathbf{x}}^{\mathrm{T}}\mathbf{Q}_{\dot{x}}^{+}\dot{\mathbf{x}} = 2315.4$. A dramatic improvement is obtained. Evidently, we should choose points located on block II to define the datum.

The above results are totally unexpected. Since we are dealing with a symmetric network, it seems logical to expect each set of four points will have the same ability to define the datum resulting in a similar level of sensitivity. The capacity of the network to detect movements was estimated to be independent to datum definition, but the example shows the advantage of one configuration (block II as datum) over the other. From an accuracy and reliability point of view each set of points defines the same network. In the example, the accuracy of the network points and their reliability does not change when switching between the two datum sets.

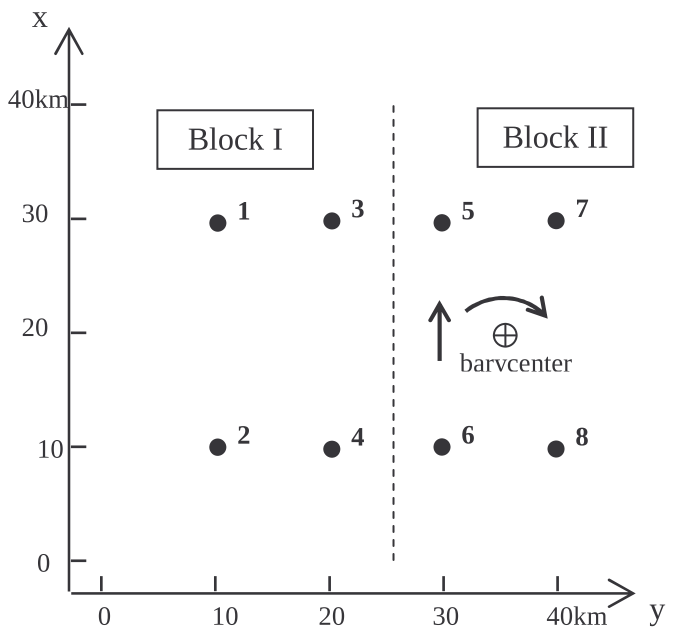


Figure 2: A two-dimensional GPS control network for monitoring crustal deformation.

7. Summary and Conclusions

The sensitivity of a geodetic network is defined as its capacity to detect and measure movements and deformations in the area covered by the network. The network points represent different areas (blocks) and the measurements allow the determination of the relative motion and deformation between the blocks. The datum of the network should provide a solid basis for defining the velocities of the points. The velocities can be viewed as a model of the deformation phenomenon. To define the characteristics of a block we should distribute points homogeneously to cover the entire area. The network sensitivity depends on the geometrical distribution of the network points. With the same number of points, different distributions lead to different levels of sensitivity. Several rules are proposed in order to define the distribution of points in a way that could improve the sensitivity of the network.

The geometric distribution of the datum points does not dramatically affect the sensitivity of the network. The geometric distribution of the object points should be further considered, but, as strange it may seem, it is important to establish the datum based on the points that define the deformed block.

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