Spectral Analysis Techniques in Deformation Analysis Studies

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SUMMARY

Analysis of geodetic monitoring records, in the type of time series, can sometimes be simple, if for instance data have a clear trend and their noise-to-signal ratio is small.

In the cases of measurements of small-amplitude, of high noise-to-signal ratios, reflecting superimposition of different signals, spectral analysis techniques can provide the best results. In this paper we review techniques which permit to identify periodicities or hysterises in time-series, and decompose them in periodic signals, even in the case data are unevenly spaced and time series short. The evaluation of a monitoring record from the Ladon Dam, Greece, is presented as a case study.

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1. INTRODUCTION

Geodetic monitoring, and especially studies of deformation of the ground and of structures is based on measurement of certain physical variables using various types of instruments. Analysis of such measurements forming time series on the basis of mathematical and statistical techniques provides answers to questions such as “what is rate of the tectonic displacement in a certain area?”, “what is the response of a dam to the filling of its reservoir?”, or “is a certain landslide stable?”. Physical measurements, however, reflect a superimposition of different signals including random errors (Fig. 1), and consequently the aim of any investigator is to separate and analyze these signals.

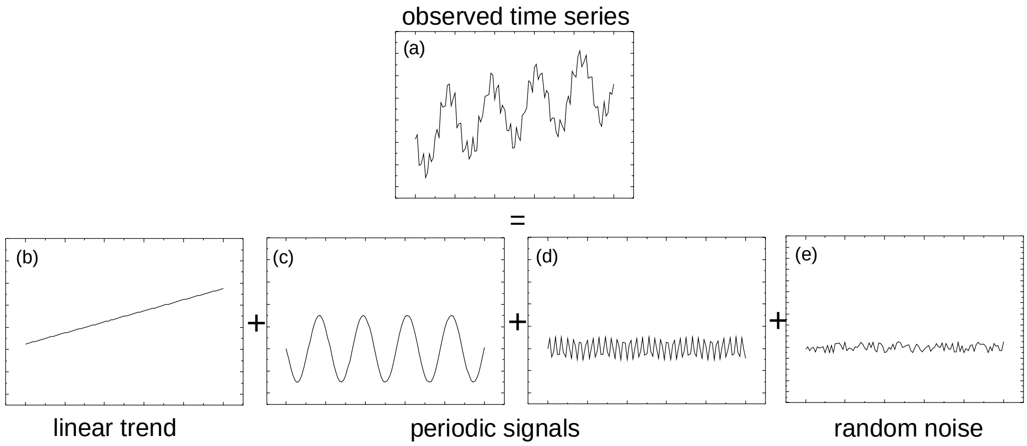


Fig. 1: The form of time series is usually complicated and does not permit easy modeling of physical phenomena. For instance, the analysis of time series (a) shows that they do not testify to a single signal but the sum of 3 individual signals: a linear trend (b), two sinusoids (c) and (d) and noise (e).

In some case, a certain signal is dominant in the time series and can be easily identified using simple techniques such as polynomial fitting or filtering using moving averages. An example is the time series of Fig. 2, representing the EDM distance change between a monitoring station on a landslide and a reference station on stable ground. A close-up of the first segment in the record (1978-1981) clearly indicates a linear trend (R^2=0.999). After this trend is removed, a nearly-periodic trend in the residuals is observed, and after the time-series is cleaned for outliers (residuals with amplitude $\geq 3\sigma$), the noise-to-signal ratio in the time series is negligible (±10mm/600mm) and hence the landslide movement can be regarded as essentially linear.

In some cases, however, the noise-to-signal ratio in the time series is high, measurements are unevenly distributed over time, and seem to reflect a superimposition of various signals, some periodic (Fig. 3); for instance stresses related to fluctuations of the level of a water reservoir or atmospheric effects in GPS measurements. In such cases simple techniques for the analysis of signals are not useful, and decomposition of measurements to various signals can only be based on spectral analysis techniques. This approach will be analyzed below.

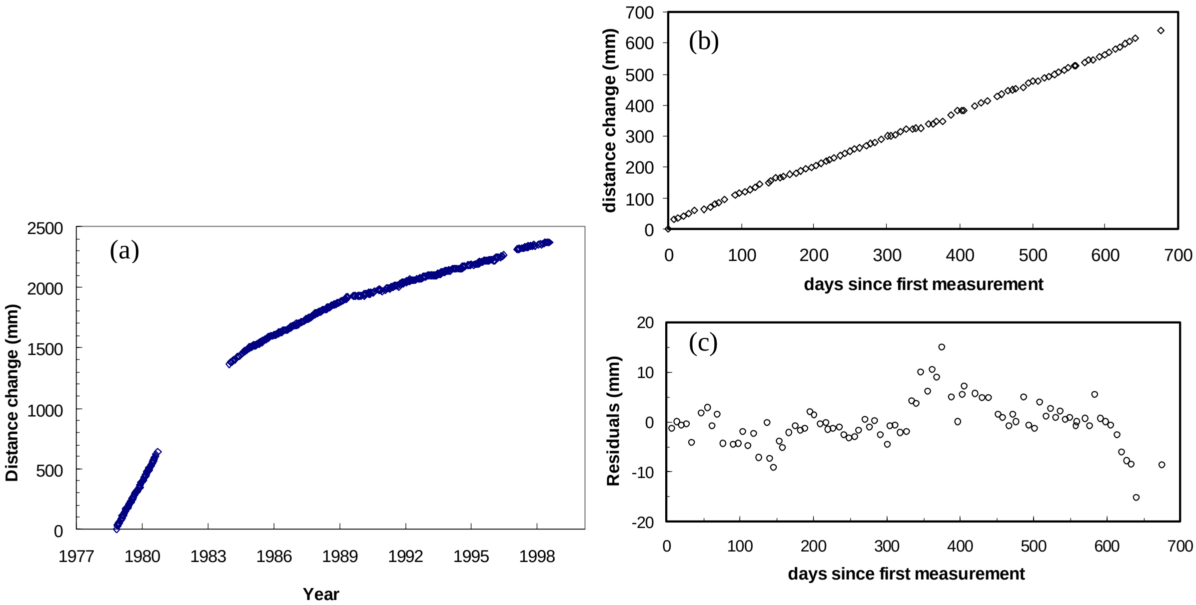
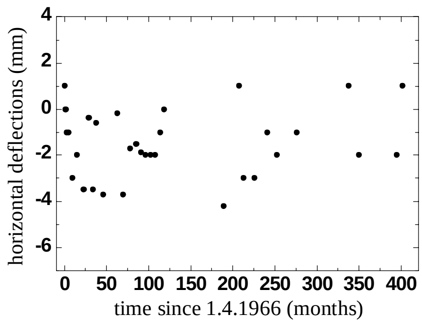


Fig. 2: (a) Time series of the distance change of a control station of a landslide in Greece from a reference station, **(b)** A segment of the time series shown in (a), first period of measurements, 10 Nov 1978 to 16 Sep 1980, 88 epochs of measurement over a period of 676 days. All measurements are included, with the exception of a blunder. A linear trend is evident (R^2=0.999), **(c)** residuals of the time series of (b) after the linear trend is removed. The standard deviation of a single observation is σ=4.5mm, but if the series is "cleaned" for the two outliers (peak values at day 376 and 641), σ=3.9mm. (After Stiros et al., 2004)

Fig. 3: Time series representing geodetically derived deflection of a control station on the crest of the Ladon Dam (Greece). Some oscillations are indicated but the small amplitude of the displacements does not permit to easily identify a possible pattern.



2. SIGNAL ANALYSIS – FIRST STEPS

When a time series in the form of that shown in Fig. 3 is to be analyzed, the first step is to identify a possible trend, and remove it (e.g. after fitting a polynomial, an exponential curve, etc., or their combination). In Fig. 1 for instance, after the removal of the linear trend, the remaining time series will consist of a superimposition of the signals of Figs. 1(c),(d),(e).

2.1 Auto-correlation

A next step is to identify whether the new time series contains periodic signals. This can be checked using the Autocorrelation function (Box and Jenkins, 1976). This function is defined by the coefficients of linear correlation (autocorrelation coefficients) between the points corresponding to the common parts of the original time series f(t) and another one, f(t+lag) for various values of lag. If the correlogram, i.e the graph of the autocorrelation coefficients versus lag has at least a quasi-periodic form, it testifies to periodicity in f(t) (Fig. 4). A requirement for these computations is that f(t) consists of equidistant data. If not, a new time series is formed using interpolation techniques.

2.2 Cross-correlation

A frequently arising problem is whether there is relationship between two variables reflected as a phase between two time series f(t), g(t) arising from measurements of two variables. For instance, if there is a hysterisis between the fluctuations of a reservoir level and the deformation of a dam or the rainfall and the activation of a landslide. Cross-correlation analysis can provide an answer to such problems.

Cross-correlation is a standard method of estimating the degree to which two series are correlated. This method is equivalent to the method of the auto-correlation but in this case data from two different time series are correlated (Box and Jenkins, 1976).

A cross-correlogram, a graph composed of pairs of numbers reflectors the linear correlation coefficients between the values of function f(t+lag) and g(t) for various values of lag is formed. A max value of c, at a lag = a, if any exists, may idicate a certain hysterisis and a possible causative relationship between f(t) and g(t) (Fig. 5).

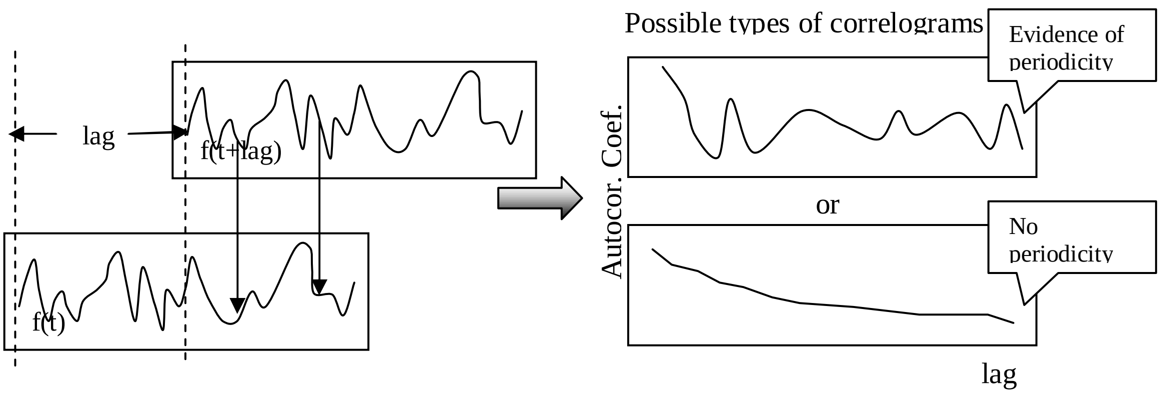


Fig. 4: The autocorrelation analysis. The correlation between the values of the time series f(t) and f(t+lag) is calculated for various values of lag. If the plot of the autocorrelation coefficient versus the lag is of oscillatory form, it reveals the presence of periodicity in f(t).

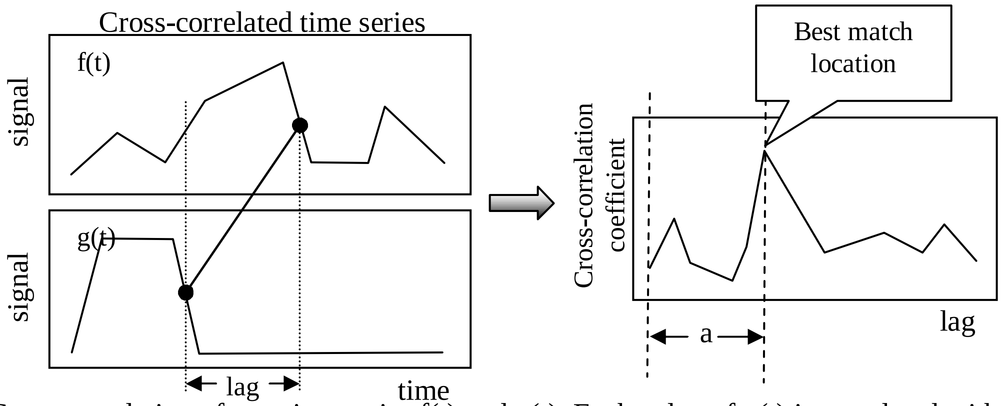


Fig. 5: Cross-correlation of two time series f(t) and g(t). Each value of g(t) is correlated with f(t+lag). The computed cross-correlation coefficients are then plotted vs lag. The maximum value in the plot indicates the time delay a, where maximum correlation is achieved.

3. SPECTRAL ANALYSIS

If a periodicity is documented, the next step is to analyze the time series in a sum of periodic signals, and estimate their period and amplitude. This can be done either using the Fourier transforms (mainly DFT) or in the case of non-equidistant data with algorithms such as the Lomb normalized periodogram (a method to analyze a signal into a sum of trigonometric terms using least square techniques) or by fitting a polynomial containing trigonometric terms to the time series.

3.1 Fourier Analysis

The spectrum of discretely sampled processes is usually based on procedures employing the Fast or Discrete Fourier Transform (FFT/DFT), the latter in the case of discrete signals. This approach to spectrum analysis is computationally efficient and produces reasonable results for a wide range of signal processes. In spite of these advantages, there are several limitations in the analysis of short data records (Table II; Kay and Marple, 1981) as well as the requirement for equidistant data. In this last case, two ways to skip this problem in the case of data, is either an interpolation or setting missing values equal to zero. Generally, such techniques are not satisfactory since long gaps in the data often produce a spurious bulge of power at low frequencies (Press et al., 1988). Additionally, most available FFT computer programs require that the number of data N to be analyzed is a power of 2, i.e. $N = 2^k$. In a different case, either some measurements are discarded, or additional zero values are added so that the new number of measurements is a power of 2; a process leading to lower quality results (Yfantis and Borgman, 1981).

3.2 Lomb Normalized Periodogram

To overcome the difficulties that are introduced when applying the FFT method to unevenly spaced data, the Lomb normalized Periodogram can be used. This technique was developed by Lomb (1976) partly based in part on earlier work by Vanicek (1969). The equation of the

Lomb normalized periodogram is identical to an equation obtained if the harmonic content of a data set, at a given frequency ω is obtained by linear least-squares fitting to the model

$$h(t) = A\cos\omega t + B\sin\omega t \quad \text{(eq. 1)}$$

(Press et al., 1988). Obviously this last method provides results superior to the FFT method for it focus on observed values, and not on sampling intervals.

3.3 Fitting a Polynomial with Trigonometric Terms

This is a technique based on least-squares method to approximate a discrete signal with a polynomial containing 2n trigonometric terms, i.e. a function F of the form

$$F = P + \sum_{i=1}^{n} k_i \sin(\frac{2\pi}{T_i} t + \theta_i) + \sum_{i=1}^{n} \lambda_i \cos(\frac{2\pi}{T_i} t + \theta_i) \qquad \text{(eq. 2)}$$

where P: a polynomial function of time, T: period, θ: phase, i=1,…n (Kay and Marple, 1981). Various functions are being tested in order to identify which one best fits the data using the trial-and-error technique.

4. A CASE STUDY: THE LADON DAM

The Ladon dam, on Ladon River, is a medium size (101.5m and 56m crest length and height respectively) concrete gravity dam, constructed between 1950 and 1955 in SW Peloponnese. The geodetic monitoring record of the Ladon Dam consists of the horizontal and vertical displacements of six control points established at the crest of the dam and the reservoir level fluctuations during the period 1968 - 2001. The time-series of the displacements make the analysis of the dam not an easy task: the time series contain just 35! values while the amplitude of the displacements is up to 7mm, though statistically significant (see Pytharouli et al., 2003; 2004). In order to investigate which is the effect of the fluctuations of the reservoir level on the dam signal analysis techniques were used.

In this case study, we focus on horizontal and vertical displacements of a control point in the middle of the crest of the dam and the reservoir level.

4.1 Formation of Evenly Spaced Time Series

Because the majority of our data was not equidistant, a requirement for methods such as the Discrete Fourier Transforms we replaced the original time series with time series containing both raw data and "pseudo – observations", the latter predicted from a 3rd order polynomial fitting. Thus 3 additional data sets were formed containing 136 values (1968 – 2001). This is the simplest way of interpolation providing an acceptable fitting to the raw data and preserving spikes.

4.2 Autocorrelation Analysis

The set of 3 time series derived from polynomial fitting was used in this analysis because of the requirement for equidistant observations (Press et al., 1992). The sinusoidal type of the autocorrelation function (Fig. 6) in all data sets revealed the presence of periodicity in our data, and made necessary the application of spectral analysis techniques in order to define that periodicity.

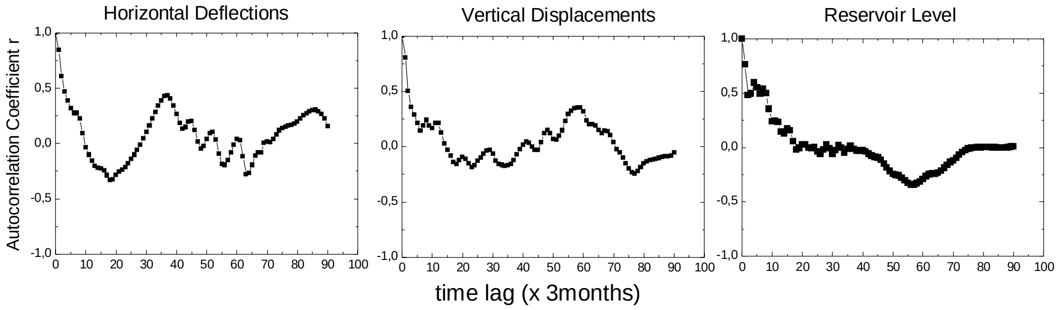


Fig. 6: Autocorrelation plots for horizontal and vertical displacements of control point C3 and of the reservoir level. In all plots the sinusoidal-type of the autocorrelation function reveals the presence of periodocity.

4.3 DFT Analysis

DFT was applied to the set of timeseries containing predicted equidistant data, as in the autocorrelation analysis. The DFT analysis identified more than one dominant frequencies (Fig. 7) in each data set. Yet among them a common frequency f = 0.0833 (1/month) was found. This frequency corresponds to a dominant period of 12 months.

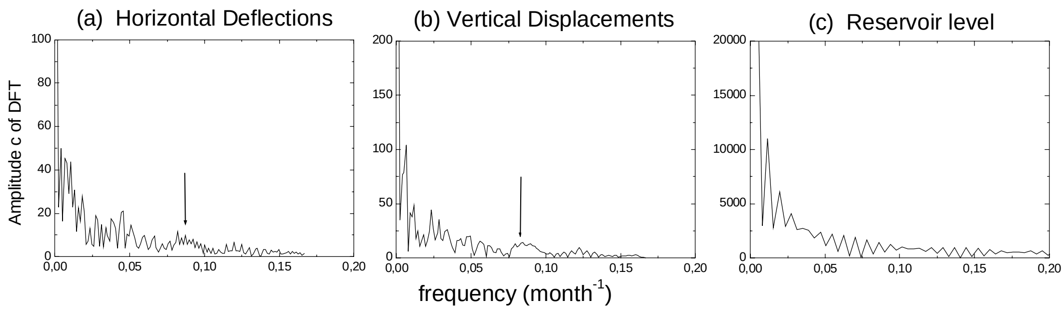


Fig. 7: Spectra plots of (a) horizontal deflections, (b) vertical displacement of control point C3 and (c) of the reservoir level fluctuations. More than one dominant frequencies are identified but among them there is a common frequency f = 0.0833 (1/month) corresponding to a period of 12months. Results are however noisy, and do not permit safe conclusions.

4.4 Lomb Normalized Periodogram Analysis

Our analysis based on this method indicated that horizontal deflections, vertical displacements and the reservoir level (i.e. independent variables) have the same dominant period equal to 12 months (Fig. 8). This result was deduced for all 3 sets of raw data analyzed.

4.5 Fitting of a Polynomial with Trigonometric Terms

We tested various functions in order to identify which one best fits our observations of dam displacement using the trial-and-error technique. All functions tested independently on the basis of their correlation coefficient provided dominant periods in the range between 11.08 –

12.56 months, i.e. close to the value of 12 months. The model with the best fit to the data had two dominant periods with values 11.08 and 12.53 months respectively (Fig.9). Such adjacent values for the dominant periods were probably the result of errors during the measurements or the calculation process and correspond to one and only value of 12 months.

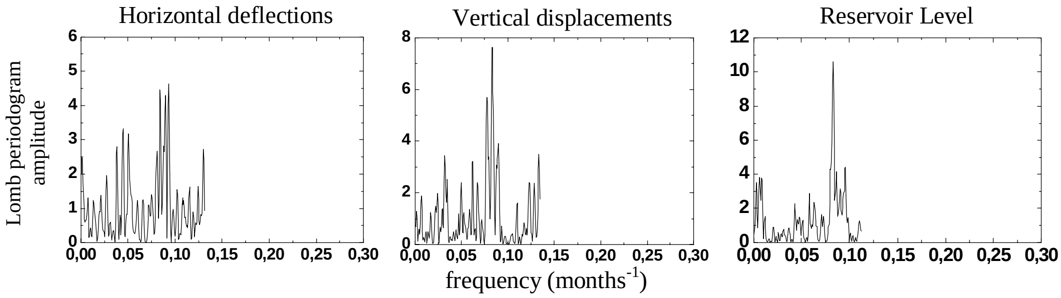


Fig. 8: Power Spectra of horizontal deflections and vertical displacements of control point C3 and the reservoir level based on the Lomb algorithm and raw data. It is shown that both horizontal and vertical displacements and the reservoir level are characterized by the same fundamental frequency f = 0.00833 that corresponds to a 12-month period. This result is far more clear than that of DFT.

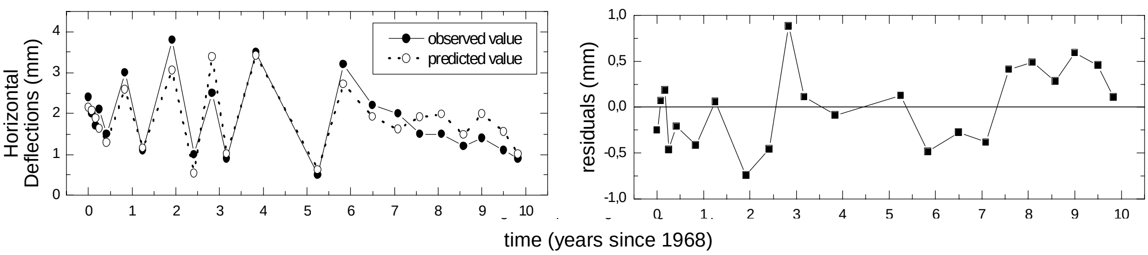


Fig. 9: Horizontal deflections of control station C3 (raw data) and predicted values of best fitting model with two periods and the residuals vs. time. The fit is quite good and the unknown periods of the model are calculated equal to 11.08 and 12.53 months. This is probably evidence of a single period of 12 months modified by measurement and calculation errors.

5. DISCUSSION AND CONCLUSIONS

Spectral analysis is a useful tool in the cases of periodic, multi-parameter and noisy time series which can be decomposed in their component signals.

In the case of the Ladon Dam, spectral analysis of a small-amplitude monitoring record without an apparent trend revealed that both horizontal and vertical displacements correspond to periodic functions with a period of 12 months, equal to the period of the fluctuations of the reservoir level. A causative relationship between hydraulic load and dam deformation was therefore inferred.

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BIOGRAPHICAL NOTES

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