Proceedings, 11th FIG Symposium on Deformation Measurements, Santorini, Greece, 2003.

AN ENGINEERING APPROACH OF A TRADITIONAL GEODETIC TOPIC THE GEOID AS HEIGHT REFERENCE AND ITS OTHER POSSIBLE USES

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Abstract

Reference points are needed in any deformation study. Geodesy contributes by measurements and provides accurate reference points through evaluation of the measurements. The duality of Geodesy as discipline that involves in common the physics and geometry of the Earth is illustrated most in Physical Geodesy. Physical geodesy deals with the study, modeling and computation of the Earth's gravity field effects aiming to map the geometry of geodetic space. The most useful engineering task is to provide accurate reference height control points by the geoid. This task involves two kinds of inverse problems: a) A non linear analytical problem due to the curved gravity space on and around the Earth b) The inverse gravity problem due to the use of gravity field data that do depend on the effect of mass-density within the Earth, that needs to be reduced! Consequently geoid undulations N may be computed with a wide rate of accuracy varying between order of cm up to some meters! This problem is most actual in continental parts where tectonic or other motions may co-exist with any local or regional deformation.

The Engineering approach of geoid as height reference starts with observed undulations via GPS and leveled /trigonometric heights. These interpolate as stochastic Earth signals. The approach avoids the use of gravity data and gives equal importance to the contribution of observations and of the stochastic interpolation model. Thus "mass-density" effects upon observed geoid signals remain intact and the pattern of interpolation errors in respect to test values is expected to depict the geographical locations where strong or discontinuous mass-density effects can by traced upon the geoid. The approach contributes to 1) improve the detailed / accurate computation of the geoid locally 2) provide objective results that may be used in relation to tectonic or deformation studies 3) contributes to the evaluation of uninvestigated areas from environmental hazard point of view at low cost.

The abundance of GPS observations all over the world for geodynamical studies when are combined with topographic heights provide "physical" geoid data about an area. These are used to interpolate a local geoid that may be evaluated in respect to any geopotential model. We present applications of this engineering approach in Greece illustrating results from a case study area around Attika where 58 linearly interpolated geoid signals depict interpolation errors that follow exactly the pattern of tectonic faults and the seismic foci of the region! The interpolated local geoid provides more detailed local features as height reference than OSU91A and EGM96 models. The result "opens" the discussion how to evaluate accurate geodetic observations in their content of information about the dynamical behavior of the Earth.

1. Introduction – Geodetic background

The Earth's surface is in permanent motion for different reasons. The motion varies at amplitudes between a few mm to some dm / year and may be expressed through deformation or kinematical models in association with time dependent observations. Geodetic observations are a well-accepted source of measured information suitable to detect motions or deformations at points on the Earth's surface or on "objects" upon it. Two sorts of geodetic points are in use for these purposes: 1) control points - representatives of a motion / deformation 2) reference points - considered as motionless. All evaluation models are composed by:

a) An *analytical model* based on a Mathematical, Physical or empirical description of the type of the process [dislocation, velocity, acceleration) to be detected
b) A *statistical model* that describes an assumption about the measuring and model errors.

Modeling involves deterministic decisions either in form of prior information or in context of assumptions. Also the type, the accuracy, reliability and the spatial /time resolution of observations refer to various reference frames and coordinate systems. For the previous reasons the evaluation of geodetic measurements for kinematics, is characterized by 2 features:

1) Certain *constraints* apply upon the evaluation model and they *deteriorate the content of information* carried within *observations*.
2) The *reference points should be assured that do not move*!

A contribution to the modeling is to detect in advance those points that are representative of the expected rate of motion in space / time and/or are "motionless" within certain time scale. The aim of this work is 1) to illustrate how *observed* geoid undulations when treated as Earth signals contribute to this task and 2) to promote the use of observed geoid in Earth kinematics and geo-prospecting as low cost explorative method. An observed undulation is a "by-product" of GPS and leveled height observations and contributes well as a complementary source of information on lateral mass-density variations within the Earth's crust independently of other geological or geophysical investigations. In this context the geoid is not "another source of gravity field data" but *rather a signal that carries information about gravity field effects that might cause ground motions.*

2. Determination and use of geoid

In engineering the geoid determination is used to connect heights **h** provided by the Global Positioning System (GPS) – to heights **H** above Mean Sea Level (Fig. 2). This "classic" problem of Physical Geodesy is studied since long. Its recent interest as active research area is due to the conversion between the two height systems that are in use. In Geophysics, its use became actual after the improvement of knowledge of the Earth's gravity field with satellite missions (e.g Kaula 1967, Parsons & Daly, 1984, Hager, 1984, Lambeck 1988). In this context geoid undulations relate with major geophysical formations. GPS-Geodesy uses the geoid in three aspects: 1) GPS points are rather sparse and so the geoid offers a way to interpolate heights. 2) Most countries use a height system related to mean sea – level so that geoid determination is needed. 3) In certain Engineering tasks the direction of flow is necessary and this is only revealed by accurate height differences δH referring to Mean Sea Level / geoid.

The “traditional” separation in Geodesy between surface and height coordinates is due to the “curved” space built by plumb – lines and surfaces of equal potential **W** of the Earth’s gravity field. This space is the *measurement space* and differs from the *model space* within which the Geodetic coordinate systems are defined (Fig.1,2). In height this difference is expressed through geoid undulation (Fig.2) and the angle of deflection of vertical. In applications one uses geodetic observations within *measurement space*, a modeled process describing in *model space* a “deterministic” aspect of a procedure or algorithm. According to the modeling assumptions, data are subjected to reductions corrections or filtering before evaluation, so an amount of physical information carried in data is lost. This occurs also in methods of geoid determination Height differences **δH** are true coordinates equivalent to potential differences **δW** in the measurement space. Height differences **δh** are model coordinates. Thus a geoid undulation operates as height “correction” between measurement space and model space, through the approximate formula (1).:

$$\mathbf{H=h+N} \qquad (1)$$

N is the geoid undulation, **h** is geometric height, **H** is leveled or trigonometric height. Eq. (1) provides raw geoid undulations that are useful: 1) For local geoid approximation by interpolation functions (local surface geoid model) 2) For testing the overall accuracy of a local geoid that has been computed by gravimetric methods. 3) As an Earth’s output signal related to inner mass-density variations in lateral and vertical sense.

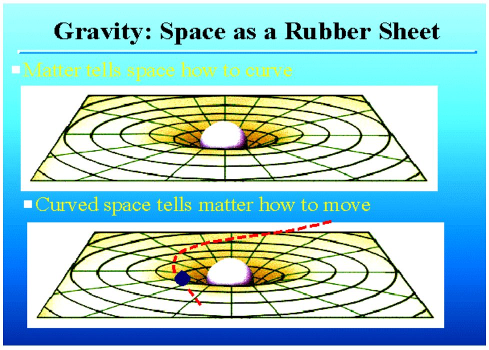

Fig. 1 The duality between geometry and physics in gravity space

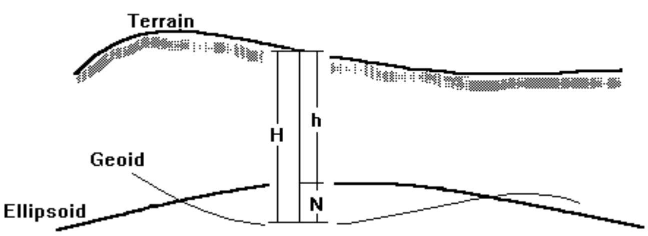

Fig.2 The definition of the two height systems in Geodesy

Observed undulations **N** can be treated as geo-signals when a lower frequency trend is removed first before to evaluate their content of information (Kahn et al. 1979) By using the observed geoid there is not loss of measuring information and no gravity or geological information is involved. This treatment contributes to geo-prospecting and opens a new field of research in the symbiosis of geodesy with applied Earth disciplines related to kinematics.

The geoid determination involves in theory the definition and the solution of various types of boundary value problems for the Earth's external gravitational potential. In practice the "classic" gravimetric approach 1) with Stokes integral (Heiskanen & Moritz 1967), 2) by solving a special oblique derivative boundary value problem on Earth's surface according to Molodenski (1962) 3) with collocation (Moritz 1972) is based on a combination of a gravity field expansion model (eq. (2)) and terrestrial gravity data and other observations, either over the entire Earth or over particular regions.:

$$N = [\, GM / r\gamma \,) \sum_{n=2}^{Mmax} (\, a / r \,)^{n} \sum_{m=0}^{n} (\, C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) \, P_{nm}(\cos\theta) \qquad (2)$$

GM is the geocentric gravitational constant, **γ** is the normal gravity {**r,θ,λ**} are spherical polar coordinates, **a** is the equatorial radius ; $\mathbf{P_{nm}(\cos\theta)}$ are fully normalized associated Legendre functions of degree n and order m; and **C, S** are fully normalized coefficients, reduced for even zonal harmonics of the ellipsoid, complete to a degree n and order m. Most needs of Geodesy are satisfied by a computed geoid through expansions at n=180 or n=360. In most approaches the undulation **N** is usually split in 3 parts as in eq. (3) in which the contribution of N_{model}. is given by eq. (2):

$$N = N_{model} + N_{\Delta g} + N_{H} \qquad (3)$$

The second term in eq. (3) represents the contribution from local gravity field data and the third term all direct and indirect topographic effects (e.g Sjoberg 1994). Computed undulations N are not complete signals because of the various modifications to gravity field data and to model simplifications / assumptions in evaluating the terms $N_{\Delta g}$ and N_H There exist also differences among model expansions that may rise up to 10 meters in N_{model} (Rapp & Wang 1993). Thus computed **N** are not a pure Earth signals.

3. The spectral configuration of the geoid

The *approximate spectral relationship* between the information per wavelength in geoid and the depth of a mass-density anomaly that generates particular geoid anomalies was introduced by Rapp (1977). Spectral ranges that associate to particular degree intervals of expansions have been studied analytically and numerically. Schwarz (1985) using existing gravity field models estimated that about 92% of the geoid's spectral information is contained until degree n= 180 of expansion. Consequently geoid "anomalies" between an observed geoid and a model expansion truncated at degree n:

$$\boldsymbol{\delta N = N - N_{model}} \qquad (4)$$

include effects of mass-density anomalies when the expansion is used in context of trend. "Anomalies" **δN** inhere the non-uniqueness of a geophysical inverse problem because any mass configuration at different depths may result to the same geoid height / anomaly. Bowin (1983) estimated the relation between the maximum depth **z** at which a point mass-anomaly generates a geoid height N at the surface of the Earth :

$$\mathbf{N = G\delta m / z\gamma} \qquad (5)$$

Eq. (5) is formed after Bruns formula (Heiskanen & Moritz, 1967 p. 85). The corresponding gravity anomaly **Δg** related to the same point mass-anomaly is:

$$\mathbf{\Delta g = G\delta m / z^2} \qquad (6)$$

The combination of eq. (5) and (6) relates **N, Δg** and **z** of a point mass:

$$\mathbf{z = \gamma\, N / \Delta g} \qquad (7)$$

If in eq. (7) computed values of N and Δg from a spherical harmonic model are put where **Δg** is:

$$\Delta g = [\, GM / r^2\,) \sum_{n=2}^{n_{Max}} (\, a / r\,)^n (\, n-1\,) \sum_{m=0}^{n} (\, C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) P_{nm}(\cos\theta) \qquad (8)$$

the depth **z** of a "source" corresponding to degree n is function of n and of the radius R of a spherical Earth:

$$\mathbf{z_n = R \;/\; (\,n-1\,)} \qquad (9)$$

The empirical law in eq. (10) connects the degree of expansion to a wavelength **λ**:

$$\mathbf{\lambda \;=\; 360 / n} \qquad (10)$$

Combination of eq. (9) and (10) gives the depth z of a point "source" as function of wavelength

$$\mathbf{z_n = r\lambda / (360 - \lambda\,)} \qquad (11)$$

λ is an arc – distance in degrees. This spectral representation expresses a global scale relation *if the non-uniqueness of the geophysical inverse problem is ignored!* Models at degree n=360 yield in principle wavelengths ~ 110 km that may be caused by "sources" maximum below ~ 20 km from the Earth's surface. The new space missions (e. g GOCE, CHAMP) are expected to improve the accuracy of models about 10 times than the "best" EGM96 model (e. g Tscherning & al. 2001). Geophysical studies about the depth z of "causative" sources are controversial. Bowin (1983) argues that longer wavelengths have origin to deep "sources" but Kahn (1977) and Lambeck (1988) claim that long-wave-length variations may be interpreted within upper mantle and lithosphere. There is no unique answer about the origin of a geoid "source" since the answer depends on approach, on data that contribute to the estimation of coefficients **C, S** and on some other factors. The engineering approach provides an objective answer through a methodology based on the spectral consideration of geoid.

4. Engineering approach with observed undulations – Demonstration of results

Observed undulations are preferred over computed or estimated ones for geo-prospecting. They carry effects of "causal" gravity sources without loss of information. Their accuracy depends on observational accuracy of heights and on other geodetic factors [e. g on GPS observation mode, datum fit). Most long-wave length effects of gravity field are eliminated in evaluating the GPS height by a low degree gravity field model, so geoid anomalies "enhance" causal mass-density effects within lithosphere. Thus geoid "anomalies" are signals from *an unknown physical system* represented mainly by the regional / local lithosphere. The content of information carried by such signals was revealed in regional / local scale by numerical methodologies that use the global model expansions in context of structural trend according to eq. (4) and as reference field in respect to degree n that the revealed information corresponds The method is based on constructing numerical spectral indices in context of a spectral consideration of geoid (e.g references)

Between 1997 –2001 experiments in Greece (Pagounis 2000, Doufexopoulou & Pagounis 1997, 1998,) and with data from Italy (Doufexopoulou et a.l. 1999) with EGM96 and OSU91A models aimed to test the content in local mass-density information in lateral and in depth sense within observed undulations and to quantify this information in terms of degree **n** of expansion. All results showed that the signal treatment of observed undulations reveals lateral and depth information on regional / local mass-density at various geographical scales. For whole Greece the revealed wavelengths are less than 440 km corresponding to degree n=90 of expansion. In Italy using 104 scattered geoid signals the gulf of Naples and the area around Venice were revealed as kinematical "provinces" (reference). The overall revealed wavelengths in Italy are less than in Greece by several decades of degrees. A most demonstrative case comes from a region 2^oX2^o around Attika. The interpolation errors of 58 signals by linear and Kriging methods with trend removal at n=90 or n=120 of models EGM96 and OSU91A distribute in the maximal size at the locations of local faults and earthquake focal depths below and above 60 km (Fig. 3 , Fig. 4, Fig.5). For whole Greece the main lateral tectonic constellation was revealed using 133 observed undulations (Pagounis 2000).

5. Concluding remarks

Observed geoid performs indeed as a raw signal of the regional mass-density variations and reveals successfully the presence of independently known lateral mass-variations in the crust without need to base on previous information about geo-features or to perform inversion. The revealed geo-information corresponds for the whole Greece between the range n=70-90. The degree varies within geographical parts. In Italy the revealed mass-density variations are closer to Earth's surface with corresponding expansion degrees between n=150 - 220.

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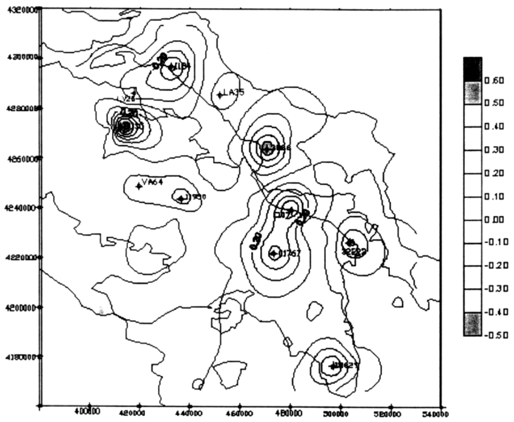

Fig. 3 The linear interpolation errors with trend of OSU91A truncated at n=90

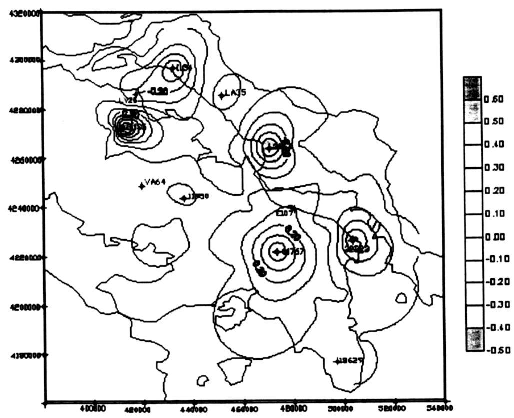

Fig. 4 Kriging interpolation errors with trend of EGM96 truncated at n=90

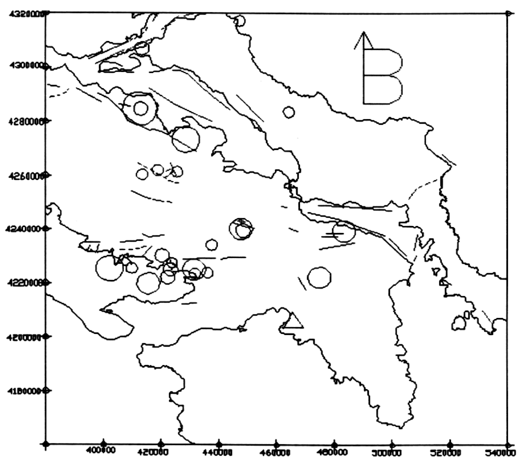

Fig. 5 Geological map of a test area with faults as line segments and earthquake focal places as circles and triangles (from. Koumbis and Apostolides, 2002).

The experiment around Attika showed that lateral mass density discontinuities were revealed geographically quite well from 58 signals. All results so far, support to use observed undulations to kinematical and deformation studies. Geoid signals can be used in evaluating regions that have probability for natural risks [e.g fault migrations, local quakes). In the first use the objective detection of lateral mass-density variations enables to select the location of control points for feasible kinematical / deformation modeling. The second use contributes to pre-select the sites that should be investigated in detail by geo-prospecting or other more expensive geo-exploration methods. Last but not less important is to use the spectral analysis of observed undulations in the context to size the geographical scale of moving regions and to select feasible reference sites to monitor deformations in large constructions (dams, highways e. t. c).

References

Apostolides , P., (2001). Measurements and applications with height methods and their geoi-prospecting capacity (in Greek). *Dipl.* Thesis NTUA, Athens, Greece.

Doufexopoulou, M., Pagounis, V., (1998) Investigation of tectonic traces upon the regional geoid without the use of gravity., *Reports of the Finnish Geodetic Institute*, 98:4, pp. 153-159.

Hahn A., Kind E.G. & Mishra D.C.;(1976) Depth calculation of magnetic sources by means of Fourier amplitude spectra. *Geophysical Prospect*.,24, pp 28-43

Kakleas & Prodromou, I (2000) Processing and analysis of height measurements*, diploma thesis* at NTUA (in Greek)

Koumbis, K, (2001) Evaluation of two interpolation methods at an observed geoid. Initially locating the geo-environmental information (in Greek). *Dipl. Thesis N.T.U*, Athens, Greece.

Milos, Sypsas, S, Triantafyllou, E (2000)

Moritz, H.; (1980) "*Advanced Physical geodesy*", Wichman.

Sjoberg, L.E (1994) On the total terrain effects in geoid and quasi-geoid determinations using Helmert second condensation method, *Rep.36*, Division of Geodesy, Royal Institute of Technology, Stockholm.

Souriau A & Souriau M.; (1982) Global tectonics and the geoid. Phys. Earth Planet Int.

Pagounis, V.N (2000). Contribution of geodetic height data processing to local geodynamics. Application in the Greek region. (in Greek). *Ph.D thesis* Faculty of Rural and Surv. Eng. NTUA, Athens

Parsons, B. & Daly, S. (1983) The relation between surface topography, gravity anomalies and temperature structure of convection, *J.G.R*, Vol. 88, B2, pp.1129-1144.

Tcherning, C, Arabelos, D, Strykowski, G. The 1-cm geoid after GOCE web file: http://www.gfy.ku.dk/~cct/banffpap.htm.